# SUPPLEMENTAL MATERIAL
## "Three-dimensional turbulence generated homogeneously by magnetic particles"

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In this supplemental material, we present movies of fluid tracer trajectories (Sec. I). Additional characteristics on mean flow and isotropy are also displayed (Sec. II), followed by details on the scaling of the turbulence level with the forcing parameters (Sec. III) and the scaling of the energy spectrum with the forcing parameters (Sec. IV).

## I. MOVIES

Movies of fluid tracer trajectories are shown for an increasing energy input (frequency $F$ and amplitude $B$ of the magnetic field) for $N = 10$ magnetic particles during 3.3 s (slow down 3 times). The fluid flow is visualized using Polyamide fluid tracers (50 $\mu$m) illuminated by a horizontal laser sheet. A high-resolution video camera (Phantom V10, 2400 × 1800 pixels$^2$ - 200 fps) records the motion of the fluid tracers. Bright dots correspond to the maximal pixel value of tracers averaged over 10 consecutive images (0.05 s). Window size = 9.4 × 8.4 cm$^2$. Note the rare events of rotating magnetic particles passing through the laser sheet. The fluid velocity is maximal in the vicinity of the magnetic particles.
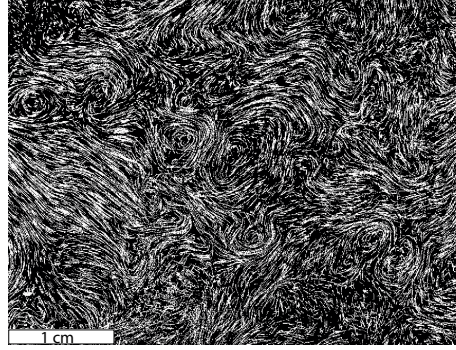


- stack5.96A20HzN10.avi: Low forcing $\sigma_u = 1.6$ cm/s ($F = 20$ Hz, $B = 137$ G),

- stack4.25A40HzN10.avi: Medium forcing $\sigma_u = 2.8$ cm/s ($F = 40$ Hz, $B = 98$ G),

- stack7.48A50HzN10.avi: Strong forcing $\sigma_u = 3.8$ cm/s ($F = 50$ Hz, $B = 172$ G).

Side and top views of an encapsulated magnetic particle (1 cm):
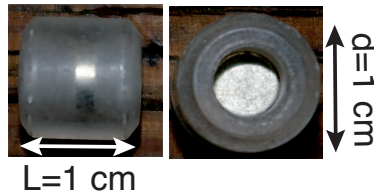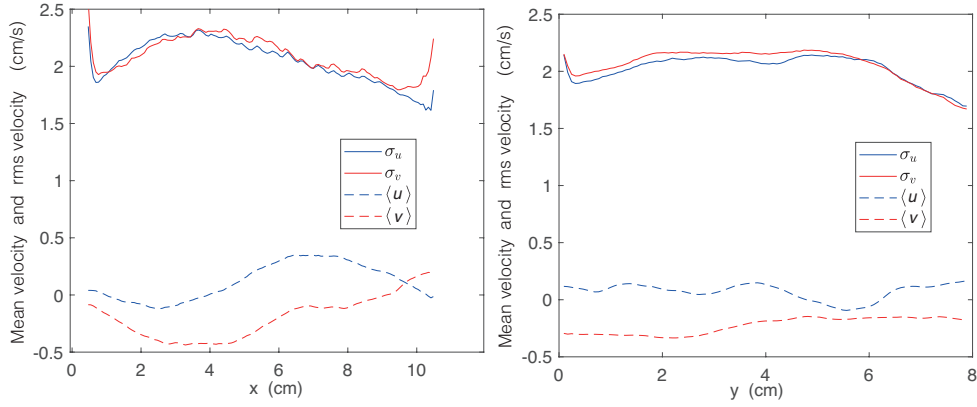
## II.  MEAN FLOW AND ISOTROPY



FIG. S1: Mean velocity fields $[\langle u \rangle_t; \langle v \rangle_t]$ and RMS velocity fluctuations $(\sigma_u; \sigma_v)$ of the fluid as a function of the coordinates $(x)$ and $(y)$ in the horizontal plane. $\sigma_i \equiv \sqrt{\langle i^2 \rangle_t - \langle i \rangle_t^2}$, where $i = u$ or $v$. The value of the velocities are averaged for 13 s. PIV measurements. The mean velocities are found to be much smaller than the RMS fluctuations (i.e., $\langle i \rangle_{t,x}/\langle \sigma_i \rangle_x < 11\%$) to be able to neglect the mean flow. $\sigma_u \approx \sigma_v$ is found also to be roughly constant far from the container boundaries located at $x = 0$ and $x = 11$ cm. The isotropy ratios are $\langle \sigma_u/\sigma_v \rangle_x = 0.96$ and $\langle \sigma_u/\sigma_v \rangle_y = 0.98$. The velocity field is thus isotropic in the horizontal plane, the domains close to the boundaries being excluded in the computations.

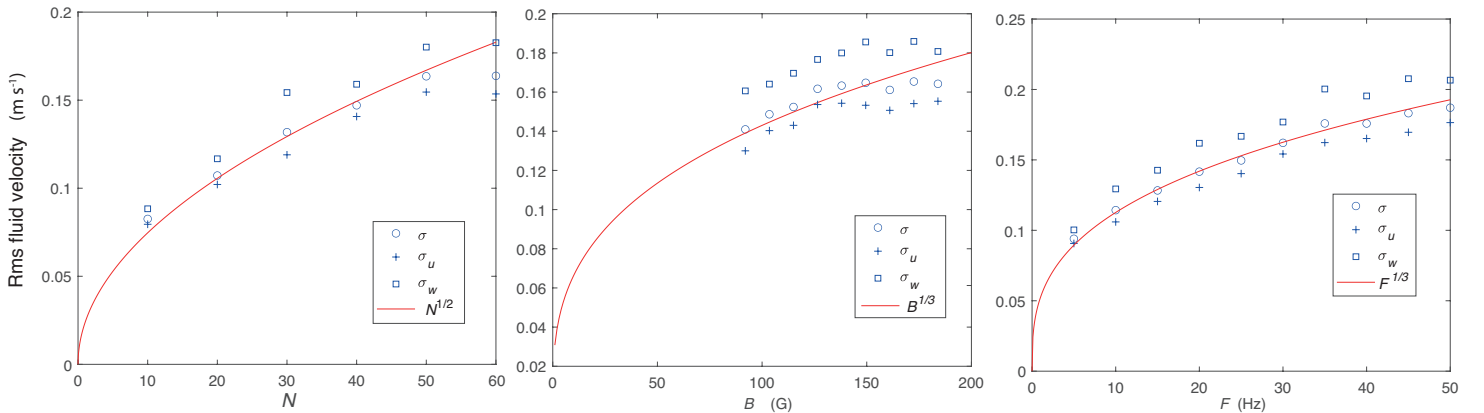## III.  TURBULENCE LEVEL WITH THE FORCING PARAMETERS



FIG. S2: Scaling of the RMS fluid velocity fluctuations as a function of the forcing parameters. Longitudinal $\sigma_u$, vertical $\sigma_w$, and total $\sigma \equiv \sqrt{(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)/3}$ RMS velocity fluctuations as a function of (a) the number $N$ of magnetic particles (for fixed $B = 161$ G, $F = 30$ Hz), (b) the magnetic field strength $B$ (for fixed $N = 60$, $F = 30$ Hz), and (c) the magnetic field frequency $F$ (for fixed $N = 60$, $B = 161$ G). Solid lines display the best fits leading to $\sigma \sim N^{1/2}B^{1/3}F^{1/3}$. The transverse velocity coordinate is not shown since $\sigma_v \approx \sigma_u$ (see Fig. S1). LDV measurements were performed at the center of the horizontal plane and a distance of 3.5 cm above the bottom of the container.

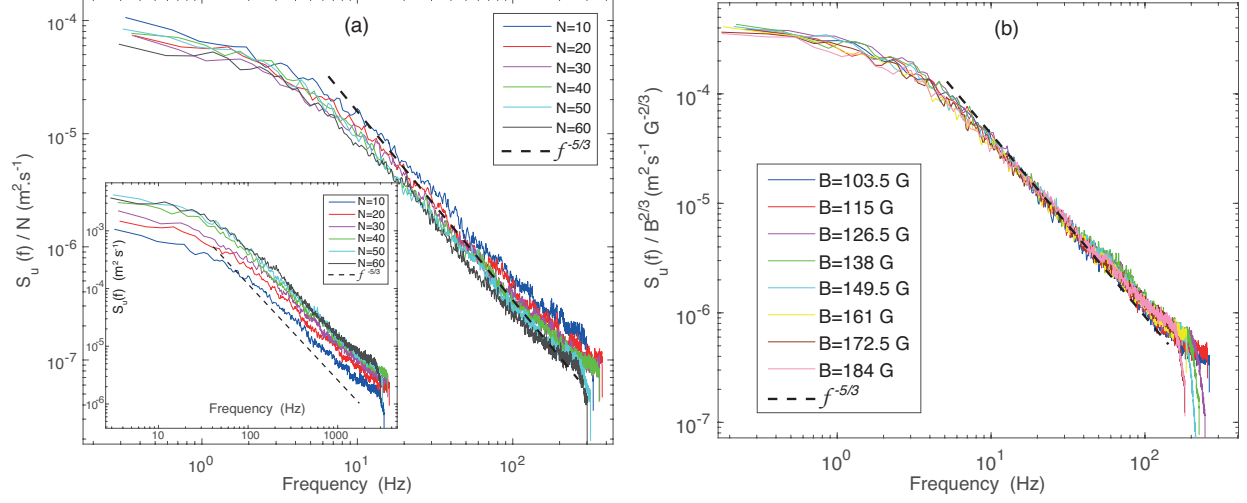## IV.   ENERGY SPECTRUM SCALING WITH THE FORCING PARAMETERS



FIG. S3: (a) Frequency power spectrum density $S_u(f)$ rescaled by $N$ for different magnetic particles $N \in [10, 60]$ (for fixed $B = 161$ G, $F = 30$ Hz). Inset: Same unrescaled. (b) $S_u(f)$ rescaled by $B^{2/3}$ for different magnetic field strength $B \in [103, 184]$ G (for fixed $N = 60$, $F = 30$ Hz). The spectra $S_u(f)$ are well superimposed when rescaled by (a) $N$ and (b) $B^{2/3}$ as expected from $\sigma \equiv [\int S_u(f,t)df]^{1/2}$ with $\sigma \sim N^{1/2}B^{1/3}F^{1/3}$ (see Fig. S2). See Fig. 3 of the main article for the full rescaled and compensated spectra. LDV measurements. Dashed lines correspond to a $f^{-5/3}$ scaling from the Kolmogorov's spectrum [3] and the Tennekes' model [29] (see text of the main article).