Energy injection in closed turbulent flows: Stirring through boundary layers versus inertial stirring

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The mean rates of energy injection and energy dissipation in steady regimes of turbulence are measured in two types of flow confined in closed cells. The first flow is generated by counterrotating stirrers and the second is a Couette-Taylor flow. In these two experiments the solid surfaces that set the fluid into motion are at first smooth, so that everywhere the velocity of the stirrers is locally parallel to its surface. In all such cases the mean rate of energy dissipation does not satisfy the scaling expected from Kolmogorov theory. When blades perpendicular to the motion are added to the stirring surfaces the Kolmogorov scaling is observed in all the large range of Reynolds numbers ($10^3 < \mathrm{Re} < 10^6$) investigated. However, with either smooth or rough stirring the measurements of the pressure fluctuations exhibit no Reynolds number dependence. This demonstrates that, though the smooth stirrers are less efficient in setting the fluid into motion, their efficiency is independent of the Reynolds number so that the Kolmogorov scaling characterizes, in all cases, the dissipation in the bulk of the fluid. The difference in the global behaviors corresponds to a different balance between the role of the different regions of the flow. With smooth stirrers the dissipation in the bulk is weaker than the Reynolds-number-dependent dissipation in the boundary layers. With rough (or inertial) stirrers the dissipation in the bulk dominates, hence the Kolmogorovian global behavior. [S1063-651X(97)11606-3]

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I. INTRODUCTION

Kolmogorov [1] was the first to emphasize in a quantitative manner the importance of the rate of dissipation of energy ε_D per unit mass in turbulent flows. In his first paper devoted to turbulence, he assumed a scale invariance of a particular kind (Eqs. 15 and 16 in Ref. [1] in conjunction with similarity hypotheses and drew the consequence that ε_D has to scale as

$$\varepsilon_D \sim \frac{L^2}{T^3} \quad \left(\text{or equivalently as } \frac{U^3}{L}\right), \tag{1}$$

where L, T, and U are the characteristic length, time, and velocity of any scale in the inertial subrange. Kolmogorov defined and applied his concepts ‘‘for sufficiently small domains in the four-dimensional space (x_1, x_2, x_3, t) not lying near the boundary or its other singularities.’’

In the following we will use a nondimensional rate of energy dissipation,

$$\beta_D = \varepsilon_D \frac{L}{U^3}, \tag{2}$$

where U and L will be associated with the large scale motion of the turbulent flow. If relation (1) is valid, β_D should be a constant independent of the Reynolds number. Kolmogorov also assumed turbulence to be locally isotropic (in space) and locally stationary (in time), the latter meaning the equality of the rate of injection of energy ε_I in the turbulence and the rate of energy dissipation ε_D (or equivalently between β_I and β_D).

Since then it has been widely *believed* that scaling (1) is valid for large enough Reynolds numbers as well as in a much broader context including a great variety of inhomogeneous flows, in fact for any flow for sufficiently large Reynolds number. However, as pointed out by Saffman [2], the direct evidence for Eq. (1) is still rather weak. Most of the existing data, such as reported by Batchelor [3], Sreenivasan [4], and Lumley [5], were obtained for turbulent flows decaying in time, i.e., without sustained turbulence production (e.g., grid and jet turbulence).

Experiments in which a steady regime of turbulence is produced in a closed cell should in principle lend themselves to measurements of the global balance of energy. In contrast to the case of decaying turbulence in these systems, the mean energy dissipation has to balance the mean energy injection, and both can often be measured independently. On the other hand, it is not *a priori* evident that Kolmogorov theory can apply here because of the presence of the containing walls and thus of boundary layers.

Closed systems were mostly investigated in two cases: the thermally induced turbulence obtained in Rayleigh-Bénard cells and the turbulent Couette-Taylor flow. The Kolomogorov scaling for the global injected energy was observed in neither of these cases. Limiting ourselves here to mechanically stirred fluids, we can recall the main results obtained in the Couette-Taylor flow in the experiments due to Wendt [6], Tong *et al.* [7], and Lathrop, Finenberg, and Swinney [8]. These papers gave results of measurements of the Reynolds number dependence of the torque applied to the rotating cylinder. All the reported results are clearly different from what would be expected from Kolmogorov theory. The total input

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power P_I being known from the torque, it is possible to deduce from it an average nondimensional rate of energy injection β_I by

$$\beta_I = \frac{P_I}{\rho V} \frac{L}{U^3}, \tag{3}$$

ρ being the density of the fluid and V the volume of the cell. The results reported by Wendt [6] and Tong *et al.* [7] exhibit a power-law dependence for the torque which, set in relation (3), yields that β_I is proportional to $\mathrm{Re}^{-0.3}$ [6] or $\mathrm{Re}^{-0.2}$ [7] a result clearly different from that expected from Kolmogorov scaling. Marcus and co-workers [9,10] adapted to the Couette-Taylor experiment a type of calculation first done by Malkus and Veronis [11] in the Rayleigh-Bénard case. This calculation takes into account the presence of walls by investigating the coexistence of two stable boundary layers with a bulk inviscid flow. Using a marginal stability analysis for the boundary layers, this approach yields a power-law dependence for $P_I(\mathrm{Re})$ of the type found in the experiments. Lathrop, Finenberg, and Swinney [8], performing very precise measurements of the torque, refined these results. Their data, obtained in a large range of values of Reynolds number, demonstrate that the Re dependence of the torque is not a simple power law. Instead they found local exponents continuously evolving with increasing Reynolds numbers. They interpreted this result using the Prandtl–von Kármán model of boundary layers. Finally, from a theoretical point of view, Doering and Constantin [12] examined the energy dissipation in a shear-driven turbulence confined between parallel walls, and found an upper bound for the global dissipation. This upper bound has a scaling corresponding to that predicted by Kolmogorov.

The problem addressed here is thus whether or not it is possible to obtain experimentally the theoretical Kolmogorov scaling for the total-energy injection and dissipation in a sustained turbulent flow in a closed cell. We will investigate this global energy problem in closed cells for two different experiments. The first is the flow between counter rotating stirrers [13,14]. The second is a Couette-Taylor flow. In both types of experiment the fluid is set into motion by moving solid surfaces. Two variants of each experiment will be examined in which these surfaces will be either smooth or equipped with platelets perpendicular to the motion.

These experimental systems being examples of sustained turbulent flows, we will check the balance between energy input and dissipation. This last aspect is addressed via independent measurement of the total energy input P_I and of the global energy dissipation P_D which will be measured, in each experiment, as a function of the Reynolds number.

Finally, in each case measurements of the pressure fluctuations will yield the intensity of the velocity fluctuations in the bulk of the flow (i.e., far from the boundaries). An independent estimate of the dissipation, in this region only, will be deduced from these measurements.

II. EXPERIMENT A

The first experimental system uses a geometry introduced for turbulent studies by Douady, Couder, and Brachet [13], and widely used since [14–16]. This system (Fig. 1) consists of a cylindrical tank with two coaxial contrarotating stirrers at the top and the bottom of the tank. Details can be found in Ref. [14].

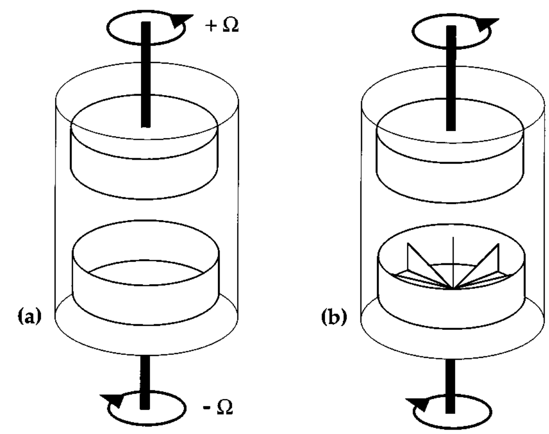

FIG. 1. Schemes of the two variants of experiment A. (a) Smooth stirrers: the two stirrers are disks with a cylindrical rim. (b) Rough or inertial stirrers: inside the rim, six blades are placed perpendicular to the disk surfaces and thus perpendicular to the rotation velocity.

The cell is a cylinder of radius R_0 with two stirrers of radius $R=0.9R_0$ at each end. The space separating the stirrers has a height h equal to R (see Fig. 1). We used two experimental cells differing in size by approximately a factor of 2. In the smaller one $R_0=8$ cm, so that its volume is $V=3$ dm^3. In the larger one $R_0=15$ cm and the volume is $V=22$ dm^3. The two stirrers were rotated in opposite directions at a frequency $\Omega/2\pi$ ranging from 1 to 6 Hz. It has been shown experimentally that this system is maximally efficient in forcing turbulence when the stirrers are counterrotating with equal angular velocities. In this geometry the main flow consists of two superposed tori which rotate relatively to each other so that they define a shear layer between them. Since these two circulations fill the whole cell, the typical large length scale of the main flow is the radius of the cell $L\sim R$ and the velocity scale is given by the peripheral velocity of the disks $U\sim\Omega R$. It is thus natural to define the Reynolds number of this flow as $\mathrm{Re}=\Omega R^2/\nu$.

Our main goal here is to check the scaling of the global rate of turbulent energy dissipation in a wide range of Reynolds numbers. We thus used the possibility of tuning the rotation velocities and used two fluids: water ($\nu=0.01$ cm^2/s and $\rho=1$ g/cm^3) or a glycerol solution (diluted with 20% water so that $\nu=0.62$ cm^2/s and $\rho=1.2$ g/cm^3). Five ranges of Reynolds number were thus investigated: $10^3<\mathrm{Re}<2.5\times10^3$, $5\times10^3<\mathrm{Re}<2.5\times10^4$, $6\times10^4<\mathrm{Re}<9\times10^4$, $1.5\times10^5<\mathrm{Re}<3\times10^5$, and $6\times10^5<\mathrm{Re}<2\times10^6$. We complemented these results with measurements of input power done by Zocchi *et al.* [15] in a similar experiment using helium gas at low temperature as the working fluid, and where Reynolds numbers as high as $\mathrm{Re}\sim3\times10^6$ were reached.

The aim here was to investigate the effect of the geometry of the stirrers. In the first set of experiments the stirrers were disks of radius R with a cylindrical rim of height $0.6R$ [Fig. 1(a)]. In the second set of experiments radial blades were fixed perpendicular to the plane of the disks and radially, so that they would be normal to the stirring velocity [Fig. 1(b)]. Two types of blades were used. In the first case there were six radial blades of height $0.25R$. In the second there were ten small ribs of height $0.04R$ only.

In the absence of blades the velocity of the stirrers is locally everywhere parallel to their surfaces; for simplicity we will call these stirrers smooth. In the latter cases the blades are perpendicular to the motion and for simplicity we will call these stirrers rough (or inertial) stirrers.

The energy input P_I was obtained through the measurements of the total power consumed by the motors driving the disks (from which the power necessary to drive the empty system was subtracted). With the orders of magnitude used for the definition of the Reynolds number ($L=R$ and $U=\Omega R$) and for the volume $V=R^3$, we define from Eq. (3) the expression for β_I as a function of P_I by

$$\beta_I=\frac{P_I}{\rho\Omega^3R^5}. \tag{4}$$

As for the global energy dissipation, it was obtained via measurements of the rate of increase of the mean temperature θ of the fluid in the system. In order to minimize the losses, the experimental cells were thermally insulated. To avoid a drift of the viscosity of the water-glycerol mixtures each experiment was started with the fluid at a fixed temperature θ_0 close to the room temperature. Then a measurement of $\theta(t)$ with a temperature probe having a resolution of $\Delta\theta=0.01°$ was done at constant time intervals, and the dissipated power was obtained by

$$P_D=C\,\frac{d\theta}{dt}, \tag{5}$$

where C is the heat capacity of the system. Again, from these measurements and Eq. (3), the nondimensionalized average rate of dissipation β_D can be deduced by

$$\beta_D=\frac{P_D}{\rho\Omega^3R^5}. \tag{6}$$

Finally it will be interesting to obtain an estimate of the intensity of the velocity fluctuations in the bulk of the fluid, far from the boundaries. It was shown in Ref. [14] that the standard deviation of the histograms of the pressure fluctuations was proportional to the square of the forcing velocity but was otherwise independent of the Reynolds number. This means that this standard deviation can be considered as a measure of the rms fluctuating velocity of the flow. We thus used pressure transducers as previously described [14] to measure such histograms and define a typical velocity fluctuation U' as

$$U'=\sqrt{(2\Delta p_{\mathrm{rms}}/\rho)}. \tag{7}$$

From the values of U' found with relation (7), and following the Kolmogorov argument [Eq. (1)], it is possible to estimate the rate of dissipation in the bulk of the flow: $\varepsilon'=U'^3/L$. so we can deduce a corresponding nondimensional rate of dissipation β_B for the bulk:

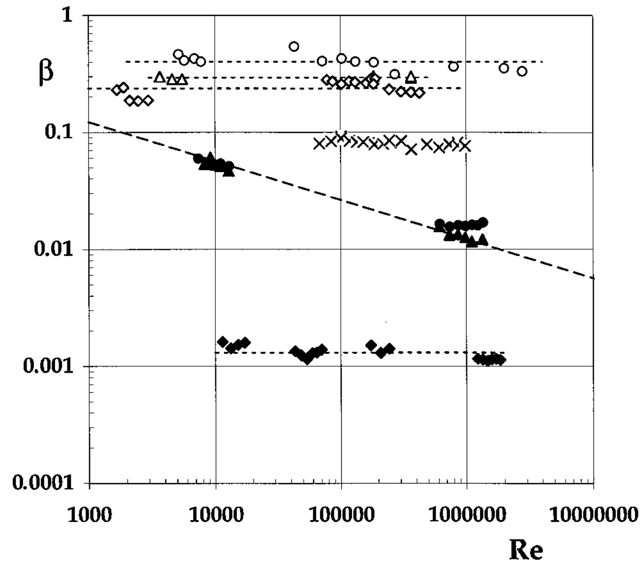

FIG. 2. Experiment A: Logarithmic plots of the nondimensional rate of energy injection β_I and dissipations β_D and β_B as a function of the Reynolds number for the three variants of the experiment. Black symbols: results obtained with smooth stirrers. Triangles (▲), rate of energy dissipation β_D; circles (●), rate of energy injection β_I; diamonds (◆), estimate of the rate of energy β_B dissipated in the bulk of the fluid as estimated from the pressure fluctuations. The dashed line shows a power law dependence proportional to $\mathrm{Re}^{-1/4}$. Open symbols: results obtained with the very rough (or inertial) stirrers. Triangles (△), rate of energy dissipation β_D; circles (○), rate of energy injection β_I; diamonds (◇), rate of energy dissipation β_B. Results are obtained with the stirrers having smaller platelets. × is the mean rate of the energy dissipation β_D.

$$\beta_B=\varepsilon'\,\frac{L}{U^3}=\frac{U'^3}{U^3}=\frac{U'^3}{\Omega^3R^3}. \tag{8}$$

Results

The results are expressed in terms of the Reynolds number dependence. The dissipation coefficients β_B and β_D can be directly compared in absolute values, as they used the same definition. But there may remain a constant of proportionality in the comparison of β_B with β_I or β_D. Figure 2 is a plot of β_I, β_D, and β_B. The comparison between β_I and β_D shows that the energy production and dissipation are in good agreement with each other. In all cases (with and without blades) the value of energy input is systematically about 25% larger than energy dissipation. This is simply due to the fact that in this experiment the heat capacity of the system was considered to be that of the fluid. As a result, the heating of the stirrers and container being neglected, the energy dissipation was slightly underestimated. The Reynolds number dependences of β_I and β_D exhibit a sharp contrast in the two cases of smooth and rough forcing.

(i) With smooth disks, β_I and β_D are found to decrease with increasing Reynolds number (Fig. 2, black symbols). In the middle range of Reynolds number they can be approximately fitted by a power law dependence $\mathrm{Re}^{-1/4}$, as shown in Fig. 2. This does not necessarily mean that this power law

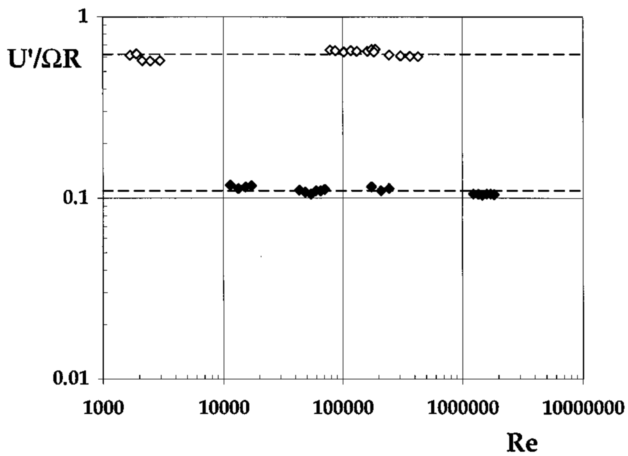

FIG. 3. Plot of the typical velocity U' in the bulk of the flow as deduced from the histograms of the pressure fluctuations [see Eq. (7)]. The black triangles (▲) are the data obtained with smooth stirrers, and the open ones (△) correspond to the data obtained with the rough ones.

would provide a good fit, had the explored range of Reynolds number been larger or the measurements more precise. In the Taylor-Couette flow (Lathrop, Finenberg, and Swinney [8]) or in wakes (Schlichting [17]) similar local power-law fits were obtained, but an exploration of a larger range of Reynolds number revealed that continuously varying values of the exponents were needed in the different ranges of Re. This behavior could well exist here also.

(ii) When the disks are equipped with blades, the input power necessary to rotate the stirrers at a given velocity becomes much larger and the dissipated power increases correspondingly (Fig. 2, open symbols). But there is also a qualitative difference: over a range of Reynolds number exceeding three orders of magnitude, β_I and β_D are now constant, meaning that relation (1) is satisfied. The Kolmogorov scaling is thus obtained globally. Additional results obtained with smaller blades show that these, though less efficient to set the fluid into motion, are sufficient to make β_I and β_D become constant in the range of large Reynolds numbers (Fig. 2).

These findings raise the question of whether or not the nature of the bulk of the turbulent flow is different with the two types of stirrings. An easy interpretation of the decrease of β_I and β_D could have been that the efficiency of the smooth stirrers to set the fluid into motion decreases with increasing Reynolds number. The integral velocity of the flow U divided by the estimated large scale velocity ΩR would then be a decreasing function of the Reynolds number. This hypothesis can be tested using the standard deviation of the pressure fluctuations and the resulting estimate of the velocity fluctuations in the bulk given by Eq. (7).

We measured these standard deviations for the first two types of stirrings (smooth and very rough) as a function of the Reynolds number. The results are given in Fig. 3. The values of U' found for the turbulence created by the rough stirrers are close to their velocity, and are six times larger than with the smooth stirrers. But the important feature is that in the latter as in the former case U' is found to be independent of the Reynolds number. This result shows that with smooth stirrers the turbulence in the bulk of the fluid, though weaker than with rough ones, has a constant amplitude. *In other terms, smooth or rough, the efficiency of a given type of stirrer to set the bulk of the fluid in motion is independent of the Reynolds number.*

Now it is possible to use β_B [Eq. (8)] to estimate the balance between the energy dissipation in the boundary layers and that in the bulk of the fluid. The values of β_B found with rough stirrers are shown by open diamonds in Fig. 2. They are only very slightly smaller than those of β_I and β_D. If, as is likely, the dissipation in the bulk accounts for most of the total dissipation, this shows that the prefactor necessary for β_B to represent the dissipation in the bulk is of the order of unity. This is also an indirect justification of Eq. (8), i.e., the use of the pressure fluctuations to estimate the rate of energy transfer directly. This is a useful result, and it will be used in a forthcoming paper [23] devoted to the investigation of drag reducing solution.

With this justification, we computed β_B for the smooth stirrers (see Fig. 2). As we found the velocity to be proportional to ΩR, the corresponding β_B was constant as in the case of rough stirrers. In the bulk of the fluid the turbulence is thus of the same nature as what would have been obtained with rough stirrers, and the dissipation in the bulk of the flow follows the Kolmogorov argument. But, in contrast to rough stirrers, β_B is seen (Fig. 2) to be much weaker than the total dissipation. The dissipation in the bulk thus only accounts for a small fraction of the total.

The turbulence created by smooth stirrers being inhomogeneous with boundary layers and a bulk central region, the results of Fig. 2 show that the dissipation in the boundary layers dominates and that it is only the decrease of the dissipation coefficient in the boundary layer which is observed. The extrapolation of the two curves suggests that at very large Reynolds number the bulk dissipation would become dominant, so that the dissipation coefficient would cease to decrease, and would stabilize at a constant value equal to the dissipation in the bulk. If the decrease of the boundary dissipation is approximated by a power law $\mathrm{Re}^{-1/4}$, this stabilization would occur for Reynolds numbers larger than 10^8-10^9. If the decrease of β_I is in fact logarithmic, as suggested by the comparison with open flows and with the Couette-Taylor case, the stabilization would occur for even larger values of Re.

III. EXPERIMENT B

The previous results obtained with smooth stirrers are similar to those of Ref. [8] in a Couette-Taylor flow. It was thus natural to check whether an inertial stirring could be obtained in this geometry too. Experiment B was thus done in a classical Couette-Taylor cell shown in Fig. 4(a). The length of both cylinders was $h=230$ mm. The radii of the outer and inner cylinders containing the fluid being $b=120$ mm and $a=75$ mm, respectively, the radius ratio was $\eta=a/b=0.625$. As in the experiment by Lathrop, Finenberg, and Swinney [8], only the inner cylinder rotated. The Reynolds number could be defined as

$$\mathrm{Re}=\frac{\Omega a(b-a)}{\nu}. \tag{9}$$

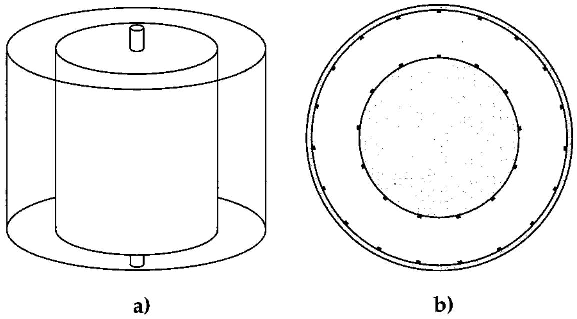

FIG. 4. Schemes of experiment B: (a) the Couette-Taylor cell with smooth surfaces and (b) the section of the system perpendicular to the axis of rotation and showing the ribs which make the walls rough.

In the first set of experiments all the surfaces in contact with the fluid were smooth. In the second, parallel ribs were glued onto both the inner and outer surfaces. These ribs were straight and parallel to the axis of the cylinders. A cross section of the cell is shown in Fig. 4(b). Since we wanted the flow to remain close to a classical Couette-Taylor flow, the ribs were chosen to be small. They were square in section, had a height of 3 mm, and were set at approximately 36-mm intervals on both surfaces.

We checked that the presence of these ribs do not change the basic flow drastically. In particular we used a very viscous solution of glycerol containing iriodine, and investigated the primary instability leading to the formation of Couette-Taylor rolls. With or without ribs it occurred practically at the same threshold, and led to a similar structure with three pairs of counter-rotating rolls along the length of the cylinder.

As in experiment A the Reynolds number was varied by tuning the velocity, and by using water or a glycerol solution diluted with 20% water. The Reynolds number range $7\times10^3<\mathrm{Re}<5\times10^5$ could thus be covered.

The same techniques were used as in the previous case to measure both the injected power and the dissipated power. They were in fair agreement with each other, but the measurements of injected power always showed more scatter, probably because of a slow evolution of the inner friction in the motors. For this reason only the dissipative powers are shown on Fig. 5. In order to obtain values of β_D from the thermal measurements, it is necessary to know the heat capacity of the system. In the present case, where a larger solid mass is in contact with the fluid, we had to measure this heat capacity separately. This was done by measuring the increase of temperature of the system when heated by an immersed resistor.

As shown in Fig. 5, with the smooth walls β_D exhibits a Reynolds number dependence. In their work on the Couette-Taylor flow, Lathrop, Finenberg, and Swinney [8] measured the global torque G exerted by the motor and gave their results in terms of Reynolds dependence of G. Introducing β_I as defined above, their adimensional torque can be written as

$$G=\pi\left[\frac{\eta(1+\eta)}{(1-\eta)^2}\right]\beta_I\,\mathrm{Re}^2. \tag{10}$$

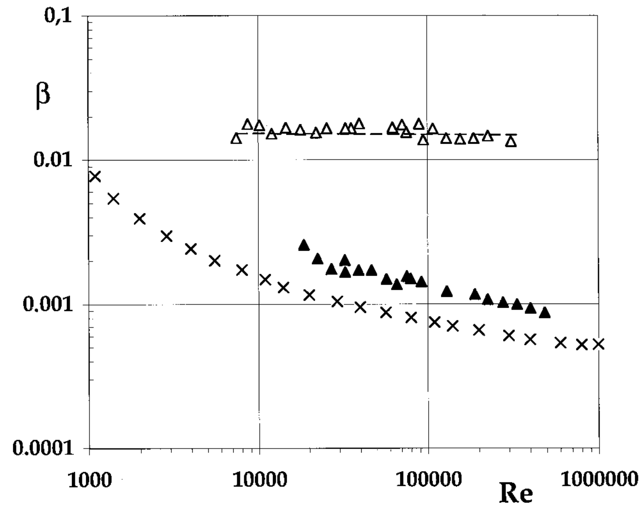

FIG. 5. Couette-Taylor experiments. Logarithmic plots of the nondimensional rates of energy dissipation β_D as a function of the Reynolds number. The black triangles (▲) are the results obtained with smooth cylinders, and the open ones (△) correspond to those obtained with the ribbed ones. The crosses (×) show for comparison the rates of energy injection β_D deduced from the data obtained with smooth cylinders by Lathrop, Finenberg, and Swinney [8].

Knowing the dimension of their cell it is thus possible to deduce β_I from their results:

$$\beta_I=\frac{G}{\pi R^2}\,\frac{(1-\eta)^2}{\eta(1+\eta)}. \tag{11}$$

Some of their points are shown in Fig. 5. Though our experiments cover a smaller range of values of Reynolds number, the Reynolds number dependence of the two experiments are in good agreement. Lathrop, Finenberg, and Swinney [8] underlined that only a local exponent could be obtained from their results. We observe the same trend in our results, but not as clearly, the temperature measurements being less precise than the torque ones. Compared to those of Ref. [8], our results are systematically shifted to larger values, either because of the difference in the geometry of the cells or because of the calibration of the heat capacity in our experiment.

When small ribs are added, the energy which has to be injected in the flow at a given value of the Reynolds number is much larger (of the order of 12 times at $\mathrm{Re}=10^5$). The important results seen from Fig. 5 is that, as in experiment A, β_D becomes essentially independent of the Reynolds number. Though our measurements, compared to those done in experiment A, covered a smaller range of Reynolds numbers, they are sufficient to show that the same conclusions can be drawn in the two cases. This type of scaling is not only that expected from Kolmogorov theory, but means that the actual dissipation is a constant fraction of the upper bound predicted by Doering and Constantin [12]. Larger dissipation rates would naturally be obtained if larger platelets were fixed onto the cylinders, and it would be interesting to see how close to the bound one can get by optimizing their shape.

IV. DISCUSSION

Our results concern statistically stationary regimes obtained in closed cells. Thus they provide a generalization of results which are well known in open flows and decaying turbulence. For comparison, we can recall briefly two classical cases: the drag of bluff and slender bodies and the resistance of pipes (cf. Schlichting [17]).

At high Reynolds number a moving body creates a turbulent wake. The energy injected into the turbulence can be deduced from the drag. In the turbulent regime, if relation (1) holds, then the drag should increase as U^2. This dependence is perfectly observed in the case of a disc perpendicular to the direction of the undisturbed flow. In this case, if normalized by ρU^2, the drag coefficient C_d (Muttray [18], Schiller [19]) remains constant over almost four decades of Re. In contrast, the experiments on flat plates, moving parallel to their main surfaces, shows that the normalized drag coefficient decreases with increasing velocities. Eventually the drag becomes constant at a value which is a function of the roughness of the surfaces.

This difference in behavior of the resistance to motion of bodies at high Reynolds numbers has led to a distinction between bluff bodies and streamlined (or slender) bodies. They differ by the way in which energy is injected into the fluid. When a disk of diameter L moves in a fluid with its surface perpendicular to the motion, large vortices of typical size L are created in the near wake just behind the disk, and then detach from it. The injection scale is thus large and well defined. In contrast, for a plate moving in its plane (at zero incidence), there is formation of boundary layers and the energy is injected into the fluid via these boundary layers, so it is not easy to define an injection scale. If the plate is rough, a transition to a constant C_d is observed at large velocity (see [17]).

The second classical situation is the flow in pipes for which a coefficient of resistance λ is defined [20,21]. This coefficient of resistance λ should be constant if relation (1) is satisfied. In smooth pipes it is in fact a decreasing function of the Reynolds number. In rough pipes this coefficient becomes constant for Reynolds numbers larger than a characteristic threshold. The larger the roughness, the smaller the threshold at which this crossover occurs.

Several theoretical models account for the behavior of both the drag of the smooth plate and the resistance of the smooth pipes. They are based on models of the boundary layers. The model due to Blasius (cf. Ref. [17]) leads to power laws of the Reynolds number. It predicts, for instance, that for the tubes λ will decrease as $0.316\ \mathrm{Re}^{-1/4}$. The model by Prandtl (cf. Ref. [17]) which provides a better fit of the experimental data gives a universal law of resistance of smooth pipes which has a logarithmic dependence on the Reynolds number and thus no power law. It was used in Ref. [8], where it provides the framework of the interpretation of the data.

V. CONCLUSION

Most recent works devoted to turbulence have concentrated on the statistical properties of local measurements such as the probability distribution function of the increment of the velocity or of the pressure in a point. Another approach has been the investigation of the coherent structures present in turbulence. On the other hand, the engineering community was always more interested in the global properties of a turbulent flow: For instance, what is the drag of a given body or the resistance of a tube? Here we have tried to revisit this problem with a view to testing the validity of the Kolmogorov model to predict such global properties as the total injection of energy or the total dissipation in a confined flow.

We have shown that the distinction between the turbulent flow created by smooth and rough surfaces is not limited to open flows, but extends to the steady regimes of turbulence in closed cells. With smooth stirring, confirming the findings of previous works, our results show that the global dissipation is weak and dominated by the dissipation occurring in the boundaries. In contrast, when the moving surfaces have platelets perpendicular to the motion, much more energy is injected into the fluid. In this case the role of the boundary layers is weaker, most of the dissipation occurs in the bulk of the fluid, and the global dissipation follows the Kolmogorov scaling. A simple interpretation is that the energy is directly supplied to the trailing vortices behind the platelets. The vortices thus typically have the size of the platelets, so that the energy injection occurs at a well-defined scale and with a well-defined velocity in the inertial range. For this reason we can call this an inertial stirring.

A general result of the present study is also the possibility of estimating directly the rate of energy transferred to the bulk of the flow by measuring the pressure fluctuations only. We found for these steady flows in closed cells that the efficiency of the stirrers in setting the bulk of the fluid into motion is a function of their roughness but is surprisingly independent of the Reynolds number. This is true even with smooth stirrers, so that in all cases the dissipation in the bulk, as estimated using the pressure fluctuations, appears to satisfy Kolmogorov scaling.

With smooth stirrers the dissipation in the boundaries is dominant and gives a drag coefficient which decreases with Reynolds number. As we found a constant drag coefficient in the bulk, this suggests that beyond a certain Reynolds number the boundary drag coefficient will no longer be dominant, so that the global drag coefficient will become constant. This turbulence in the boundaries was shown to exhibit the Kolmogorov scaling at the level of the second-order statistics [22], but it is conceivable that the existence of different regions in the flow could have an effect on some of the other statistical properties of the turbulent flow. It should be particularly interesting to see whether or not the intermittency of the velocity signals is dependent on the type of energy injection. A forthcoming article (Cadot, Bonn, and Douady [23]) will be devoted to a discussion of the drag reducing properties of diluted polymers, as observed in turbulence generated by the two types of stirrings, respectively.

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