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## Reply to the Comment by N. Fathi et al.

S. COUVREUR and A. DAERR

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Reply

## Reply to the Comment by N. Fathi et al.

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published online 1 December 2014PACS 47.20.Ma – Interfacial instabilities (*e.g.*, Rayleigh-Taylor)

PACS 47.55.np – Contact lines

PACS 47.20.Qr – Centrifugal instabilities (*e.g.*, Taylor-Couette flow)

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In a recent paper, we investigated the stability of a rivulet of water flowing down a vertical glass pane. This system exhibits various flow regimes for increasing flow rate, as described by Nakagawa [1] for water on PMMA: at very low flow rate water runs down in distinct drops. As the flow rate is increased, one observes first stationary, pinned rivulets (straight or meandering), then non-stationary streams meandering and breaking up, and at higher flow rates “restable” (Nakagawa) straight streams. Our aim was to determine why straight rivulets become unstable and start meandering. In total wetting conditions, where there are no pinning forces, we had already shown [2] that anisotropic friction due to the contact lines leads to an inertial instability. In partial wetting the pinning of the contact lines suppresses the linear instability. The instability still exists, however, because a flow rate increase will eventually increase the contact angle beyond its maximum static value, at which point the contact line recovers its mobility. The main result of our paper is to show experimentally that the critical flow rate at which this happens can be predicted by analysing the initial rivulet shape, specifically its width and roughness.

In their Comment, Fathi *et al.* describe experiments that, they claim, disagree with our findings. Although their system behaves differently in some respects (maybe due to different wetting characteristics), we do not feel that the details provided contradict our observations on the destabilisation of a straight rivulet. Indeed Fathi *et al.* agree that a “ $Q$  [flow rate] variation can [...] destabilize a rivulet”, and explaining the required variation is precisely the question addressed in our paper. The central difference of the experimental systems seems to lie in the evolution *after* meandering has started: we notice that meanders eventually become stationary, whereas Fathi *et al.* observe that meandering resulting from a flow rate change is only transient<sup>1</sup>. While it is quite interesting

that there exist systems differing in their long-term dynamics, this has little to do with the point we make in our paper, *viz.* that straight rivulets become unstable as the flow rate is increased to a critical value entirely determined by the spacing and roughness of the rivulet’s contact lines.

Fathi *et al.* see a potential for misreading the notations in our paper, notably in fig. 4 depicting a transverse cut through the rivulet, *not* a top view of the rivulet path. The revisited fig. 1 hopefully dissipates any ambiguity.

Fathi *et al.* criticise two points in our simple model for the transverse rivulet profile: that we assume that the cross-sectional area remains constant when the rivulet deforms, and that a cubic can be a reasonable approximation. For typical heights ( $\sim 1$  mm) and velocities (several 10 cm/s) of the stream, the viscous boundary layer develops over a distance of several 10 cm. The fluid velocity will hardly change when passing through a perturbation which

<sup>1</sup>Fathi *et al.* maintain that meandering requires flow rate fluctuations, and suspect our constant level tank of producing more noise than theirs. After carefully reading the description of their tank we do not see what physical mechanism could explain this. Here are some details and estimations concerning our tank ( $\sim 20$  cm  $\times$  20 cm  $\times$  20 cm), mounted 3 m above the nozzle producing the rivulet with in- and outlets in opposite walls. Water is constantly flowing over one of the tank walls, 20 cm wide. As no waves or ripples are visible at the free surface, we can take an upper bound of height fluctuations of 0.3 mm, meaning that relative pressure fluctuations at the nozzle are  $< 10^{-4}$  (outlet and free surface are equally far from the inlet). This seems negligible compared to the amplitudes used in the only perturbation experiment published so far by Fathi *et al.* [3,4], where a valve would reduce the cross-section by  $\sim 20\%$  for a duration of 0.1 s every second. Fluctuations could, of course, also arise in the tube between tank and nozzle. This tube has an inner diameter  $d = 12$  mm, so that at the highest flow rates the flow remains in the laminar range ( $Re = Q/dv < 200$ ). In Fathi *et al.* the flow rates are up to four times higher and the diameter is four to six [4] times smaller, so that turbulent fluctuations in the tube feeding the rivulet are much stronger in their set-up. In any case Fathi *et al.* provide no quantitative measurement, in support of their argument, of the fluctuation amplitude that is required to affect the flow.

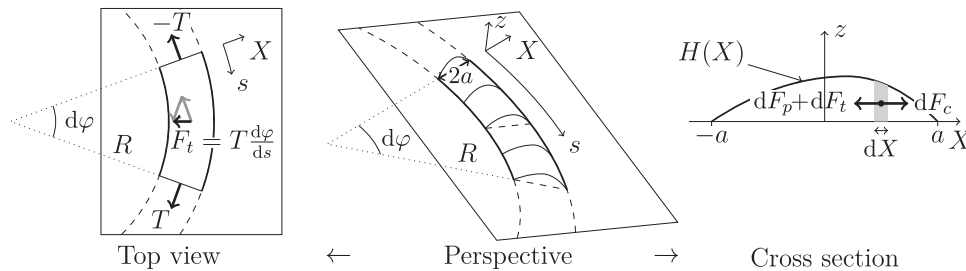


Fig. 1: Sketch clarifying the notations of the original paper. The inclined plane in the middle shows a portion of a rivulet, whose path on this plane locally has a bending radius  $R$ . We use a local coordinate system where  $s$  is the arc-length,  $X$  is orthogonal, in-plane, to the rivulet path, and  $z$  measures the distance to the substrate. The top view on the left shows that the effective line tension  $T$  (stemming from surface tension and pressure integrated here over the whole rivulet cross-section) results in a centripetal force per unit length  $F_t \propto T/R$  normal to the rivulet, along the  $X$ -direction (we neglect variations of the norm of  $T$  along  $s$ ). The sketch on the right shows a cross-section through the rivulet, where  $H(X)$  denotes the height of the free surface above the substrate and  $\pm a$  are the positions of the contact lines. We consider the forces per unit rivulet length acting on a fluid element of width  $dX$  of the rivulet. The curvature of the free surface,  $\sim H''(X)$ , increases towards the outer side of the bend (here to the right,  $X \rightarrow +a$ ), resulting in a Laplace pressure gradient which produces an additional centripetal force  $dF_p$ . In the fluid reference frame and in a stationary bend, the sum of capillary forces  $dF_t + dF_p$  compensates the height-averaged centrifugal force  $dF_c$  when the secondary flow (Dean recirculation) due to the vertical velocity gradient is neglected.

is typically smaller, so fluid flux conservation does imply an almost invariant cross-section. The rivulet profile in our model is specified by four parameters: three boundary conditions for the differential equation, plus the parameter  $\alpha$ . Two are given by the condition  $h(-1) = h(+1) = 0$ , and we take the cross-section and the contact line force  $F_r = \gamma(1 + h'^2)^{-1/2}|_{-1}^{+1}$  as two additional physical parameters (related to flow rate and inertia) that select the profile. By contrast, Fathi *et al.* arbitrarily choose  $(h', h'')|_{x=-1} = (1, -0.5)$  as additional conditions when comparing the full solution and the cubic, unsurprisingly finding a bad match. Figure 2 shows that the cubic having the same cross-section and pinning force is close to the full solution. A forthcoming paper will compare our model to experimental measurements of the rivulet deformation.

Finally, Fathi *et al.* question the RMS measurement of the curvature because of the potential dependence on cut-offs. These can play a role for extremely flat distributions, which is not our case. Furthermore the only regularising operation in the calculation of the curvature of the rivulet path is the use of a smoothing window on the stream deviation whose size is the rivulet width, *i.e.* the smallest length scale for which it makes sense to define a rivulet curvature. This effectively cuts off curvatures above around  $C_{\max} \simeq 200 \text{ m}^{-1}$  (Birniir *et al.* [3] identify a similar limit for their spectra). This value is much higher than our typical results (10 to  $30 \text{ m}^{-1}$  in fig. 3). There is no lower cut-off. We are thus confident that the RMS curvature is a meaningful quantity.

It seems that Fathi *et al.*'s claim that meandering results from fluctuations only is the result of a hasty interpretation of the experiments. Indeed we agree that over the whole range of flow rates where meanders are observed, the meandering rivulet can be straightened by interrupting and restarting the flow, or by manually re-routing the rivulet along a straight flow path with a cotton bud as in

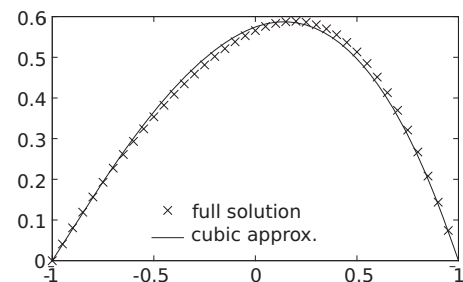


Fig. 2: The full numerical solution picked by Fathi *et al.* ( $\alpha = 17.2$ ) is approximated to less than 5% by the cubic having the same cross-section and pinning force ( $A = 0.912$ ,  $\theta_s = 1.149$ ).

our paper. This does not contradict the observation that a straight rivulet of given width becomes unstable above a critical flow rate. That stationary meanders can be destroyed by a flow rate increase is seen by the co-authors of Fathi as an indication that the substrate was dirty [3], but our paper offers a simpler explanation: meanders are metastable, and coexist with straight streams. In any case we hope that Fathi *et al.* will persist in investigating the role of flow rate fluctuations, and we look forward to new data.

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