FORMATION OF SANDPILES, AVALANCHES ON AN INCLINED PLANE

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Abstract. We present two simple experiments on granular flow. Though the notion of pile angle is common, it appears that there can be many different angles depending on the experiment [1, 2]. Our first experiment also shows, in a conical geometry, that the dynamics of the flow leading to these angles depends strongly on the density of the pile and presumably on its internal structure, in contrast (surprisingly) to the final profile. Our second experiment studies the hysteresis between the static and dynamical angle in the case of a thin layer of beads on a rough inclined plane. Between these two angles, an avalanche is amplified while going down, and we observe that it grows lateraly leaving a triangular track, whose opening angle increases as the plane inclination approaches the static angle. Before reaching this limit, the avalanche starts to propagate upwards.

1. Transients leading to the formation of a pile

The experimental setup consists of a hollow cylinder resting on a disc of slightly greater diameter. The whole is placed on top of a balance. We fill the cylinder with sand or glass beads (250–425 μm) in different ways. After having carefully removed the excess sand above the cylinder, we abruptly pull the cylinder up vertically and look at the flow with a CCD-camera.

The main feature of this flow is its strong dependence on the initial compactness. We use two simple methods to obtain a low or a high compactness in the initial state. The first one consists in pouring sand quickly from a small height at the center of the disc. Although this is a very crude method (probably not homogeneous), it provides very low densities ($\nu \approx 0.58$). The second technique uses two sieves which are some 15 cm apart and onto which we pour the sand from an upper funnel. The grain bounces result in a homogeneous downfall of the grains with a high velocity and a small rate. This 'rainfall method' yields high densities of $\nu \approx 0.65$, the random dense packing limit for monodisperse spheres. Tuning the various distances between the funnel, the grids and the sand layer varies the density continuously [3].

1.1. LOOSE CASE

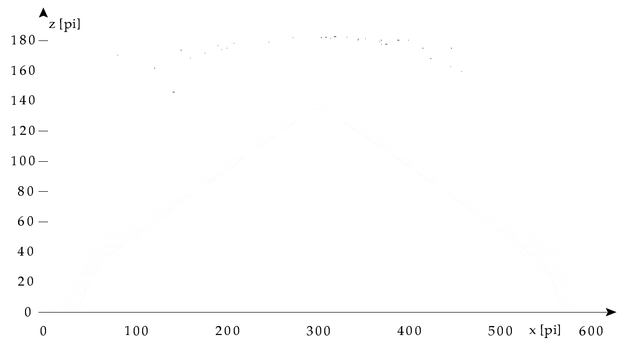

Figure 1. Series of profiles showing the formation of a pile for loosely packed glass beads, $\nu = 0.58$. The initial height is 47 mm, the diameter of the support (horizontal line) is 142 mm. The time lapse between consecutive profiles is 0.02 seconds. The flow starts with a fracture which takes the 'corners' down, but right from the beginning the tip is very rounded. Some profiles close to the end of the flow have been omitted to show that the final profile is foot-curved while the intermediate states have a well defined slope.

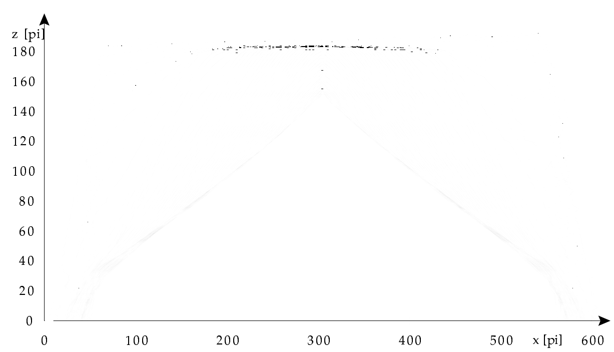

Figure 2. Formation of a pile, with the same setup as in Fig. 1, except for $\nu = 0.65$ initially and a time lapse of 0.04 s between profiles. There is a clear fracture at an angle corresponding to an active Coulomb-yielding of the material. A little later we observe a sudden change in the slope which looks like a second fracture. The profile is then characterized by the existence of two distinct regions: the upper steep region slowly moving inwards, and a left lower region at the angle of the final pile. As opposed to the loose case, the center region does not move before being reached by the steep front.

For a low initial compactness, we observe a flow as depicted in Fig. 1. Upon removal of the outer wall, part of the material falls off the disc almost freely. A tenth of a second later (roughly the time for a free fall over the height of the pile) we are left with a cone whose angle simply shows an exponential relaxation towards a constant value. The summit is rounded from the very beginning showing that the flow instantly extends to the center region. This and the rapid decrease in height are strong indications of a great depth of the flow in this loose case. Fig. 1 shows that the last part of the flow is the formation of rounded feet from the nice converged slope. Fig. 1 shows that rounded feet appear in the last part of the flow, well after the slope has converged.

1.2. DENSE CASE

For a high initial density we observe a very different flow (see Fig. 2). There is a rapid fall of the 'corners' as in the loose case, but it can now be more precisely interpreted as an active Coulomb-yielding of the material. In addition, there appears to be a second fracture a few moments later, the cause of which is unclear. But the main difference to the 'loose' case lies in the flow regime after the second fracture: While the center region remains perfectly still, the flow occurs only through an erosion at a great angle. As this slope moves inwards, it leaves behind a cone with a much shallower slope, close to the final one (see Fig. 4).

Using coloured sand, it can be seen that the flow in the two regions is very different. The grains at the surface in the upper high-angle part

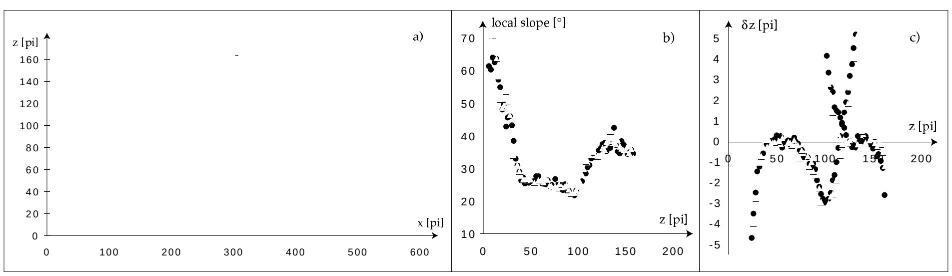

Figure 3. (a) An intermediate profile in the dense case showing the two distinct regions. (b) The local slope as a function of the height. Triangles represent the values for the left side of the profile, circles for the right side. After a decrease of the slope corresponding to the free fall off the base, we observe a first *plateau*, a transient and then a second *plateau* at a higher angle. These *plateaus* show the simultaneous presence of the two regions of different slope. (c) The deviations from the two linear fits derived from (b) as a function of height. The kink near the transition ($z \approx 100$) is a hydraulic jump from a thin high velocity (accelerated) layer of flowing grains on the large eroding slope to a thick small velocity flow rubbing on the final remaining cone.

accelerate on their way down, in contrast to the lower region where they flow at a strongly reduced, nearly constant velocity. The abrupt change in the flow regime induces the formation of a 'hydraulic jump' (see Fig. 3). This difference tells us about the physics of the two regions. In the upper region, the flow seems to be limited by erosion or the ability of the beads to unlock from their tight packing. The flowing grains seem to hardly interact with the underneath grains once they are in motion (nearly free fall). In the lower part, the flow occurs at a smaller angle and on the motionless particles of the final pile, with a lot of friction.

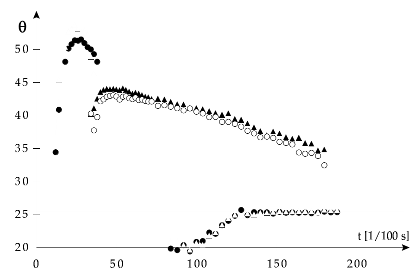

Figure 4. Evolution of the slopes for dense packings. The upper set of points corresponds to the steep region, and the lower ones to the shallow outer region. Triangles and circles describe the right and left sides, respectively. The first fracture leads to angles of the order of 52°, and is followed closely by a second fracture at $t \approx 0.35\,\mathrm{s}$ at an angle of 43°. Note that for the large upper slope there is a nearly constant angle difference (2–3°) between right and left, which may originate from a slightly asymmetrical preparation, while the second slope leading to the final profile is perfectly symmetric.

The evolution of the angles in the two distinct regions is shown in Fig. 4. Contrary to the loose case, the dynamics of the flowing angle only shows a small continuous decrease. Due to geometrical constraints, the full dynamics was not observed here (too large ratio height/diameter). But from other experiments it seems that there is a sharp final decrease of the angle to the final one (like an inversed bifurcation). As in the loose case, the final angle is reached very quickly, much before the flow has completely stopped.

A remarkable observation in the dense case is the constant, though small, asymmetry in the profiles between the right and the left sides (see Fig. 4). This indicates a strong sensitivity of the flowing regime to the intial preparation of the pile: in this particular experiment, the rainfall method had led to a final height slightly larger on the right than on the left. Initially to check our method of preparation, another experiment was made with a compact sample obtained through vibration. The vibration was so strong that it led to convection. We found an even more pronounced asymmetry of the transient, with a constant angle difference of more than 10° (data not shown). Following the model proposed in ref. [4], this effect could be ascribed both to a density higher at the foot of the convective cone than under its tip, and/or to a structuration inside the convecting heap with internal slip cones going down to the center of the heap.

1.3. DISCUSSION

We presented only the two extremes, but varying the initial density of the pile gives a continuous transition from the dense to the loose regime. The flowing transients show a strong dependence on density. The asymmetry of the transient profiles also reveal a strong sensitivity on internal structure (or fabric tensor). It is then very surprising to obtain always prefectly symmetrical final profiles and that they differ little between the dense and the loose case. A starting point to interpret this phenomenon is to recall that the slope is always better defined during the flow than in the final state. In the loose case, the flow occurs deep in the pile, and the final evolution of the profile can be seen as a freezing of the deep flow. However, the freezing layer is not supported at the limit of the disc. It will therefore continue flowing, and erode up to a larger angle where it is locally stable (see Fig. 1) [5]. By its motion, the freezed layer has forgotten the structure of the pile, so that it is natural for the final profile to be perfectly symmetrical.

In the dense case, the flowing layer is much thinner in the eroding part, but enlarges on the final cone. There is thus probably the same boundary effect in the feet, and the same structure wash-out in the final freezed layer. However, even if we assume that the thickness of the freezed layer is negligible, especially compared to the strong asymmetry observed, there can be another mechanism: when the high velocity/angle flow hits the underneath and hereafter fixed grains of the final cone, there is probably a local destruction of the original pile structure. The final cone has thus a surface which can be a mixing of a freezed flow and a reorganized layer.

2. AVALANCHES ON AN INCLINED PLANE

Thin layers of particles can remain perfectly stable on a rough, inclined plane at angles well above the classical angle of repose θ of a pile [6]. According to Ref. [6] the rough surface prevents the material from dilating, increasing θ, and this effect extends over some grain diameters inside the sand layer. This can be expressed as a height-dependence of both the repose angle θ_s and the dynamical angle θ_d (at which moving material settles). It is clearer however to think in terms of angle-dependent heights (see Fig. 5): for a given plane angle φ, there is a maximum thickness $h_s(\varphi)$ for a static layer to remain immobile, and a second, slightly smaller height $h_d(\varphi)$ at which a flowing layer freezes. The aim of this experiment is to study the hysteretic behaviour in the gap between the two static/dynamic values.

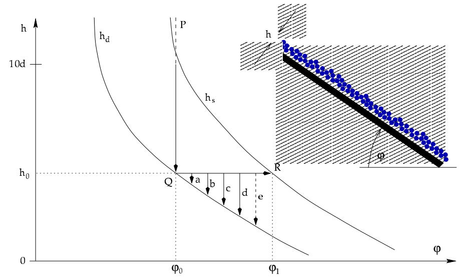

Figure 5. This diagram sketches the principle of our experiment. The two curves indicate the maximum height of a layer which is dynamically or statically stable at a given plane angle φ. We pour glass beads on the inclined plane and obtain a layer of uniform height $(P \rightarrow Q)$. We then increase the tilt angle of the plane and enter the metastable zone between Q and R. If we trigger off an avalanche (a to e), the layer thickness behind the avalanche is decreased by an increasing amount.

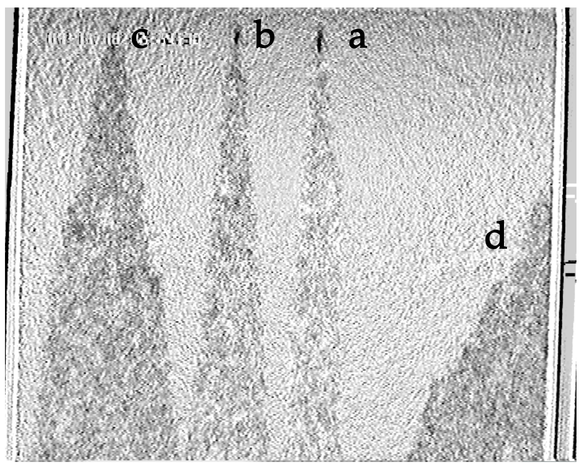

Figure 6. Image difference before and after the passage of four avalanches (a to d) at regularly increased plane angle ($\Delta\varphi = 0.25°$, $\varphi_0 = 30°$). The tracks are clearly getting deeper and deeper as can be seen from their darker aspect (white beads on black velvet). The remarkable point is the lateral propagation of the avalanches along well defined angles. The opening angles are increasing continuously from a to d. The avalanche e (from figure 5, not shown in this figure) propagates uphill, which can be interpreted as an opening angle larger than 180°.

The plane is covered with velvet cloth which is rough and shock absorbing. We tilt the plane to an angle φ_0 and pour glass beads abundantly at the top. The moving beads leave behind a layer of thickness $h_0 = h_d(\varphi_0)$. If we put a small extra amount of grains on this layer, the perturbation will move downwards without gaining mass, leaving the layer height unaffected [7]. If we increase the tilting angle to φ_x however, we enter the hysteretic zone. Any perturbation going down will now be mass-amplified, because the layer thickness will decrease to the new dynamically stable height $h_d(\varphi_x)$.

The interesting point is the lateral propagation of these avalanches. An avalanche triggered off in the hysteretic region does not only increase in mass, but it also grows laterally (see Fig. 6). The trail is triangular shaped. Such triangular tracks are observed in nature: they are known as 'loose snow'-avalanches [8]. In our experiment, the opening angle is close to zero for $\varphi \approx \varphi_0$ and increases as $h_d(\varphi)$ departs from h_0. As the opening angle increases, the frontier of the trail becomes more and more irregular and sensitive to small variations of the layer thickness. The avalanche can then start to move up and down irregularly. At a given value of φ, the front starts to move upwards in average with a small velocity. This can be interpreted as an opening angle which exceeds 180°. The sticking point is that this uphill motion appears at a tilt angle which is still smaller than the critical static angle for this starting height, φ_1. The uphill velocity increases as $\varphi \to \varphi_1$. At φ_1, any avalanche front is hardly ever observed: the whole layer starts to flow suddenly for a perturbation as tiny as a local grain rearrangement.

In our opinion, this lateral propagation reveals how a surface grain is supported by its surroundings. At the dynamical height h_0, it is supported only by the grains beneath it and closer to the plane. When approaching the static angle φ_1 it is more and more supported by its surface neighbours. When looking carefully we see that the lateral (upward) propagation of the avalanche flow is due to the lateral (upward) grains being no longer supported, and then starting to fall. This mechanism seems *a priori* different from the explanation of the uphill motion proposed in Ref. [9] based on the competition between the downward motion and an isotropic diffusion.

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