Expressions for mass and momentum conservation:

$$\partial \rho / \partial t + \operatorname{div}(\rho \boldsymbol{v}) = 0 \quad \text{(continuity equation)}$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}.\overrightarrow{\text{grad}})\boldsymbol{v} = \rho \boldsymbol{g} - \overrightarrow{\text{grad}}\, p + \eta \Delta \boldsymbol{v} \quad \text{(Navier-Stokes equation)}$$

In cylindrical coordinates

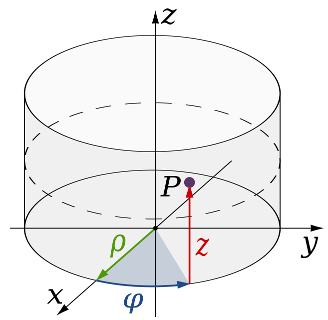

illustration: Wikimedia/Cylindrical_Coordinates

continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial(\rho u_\varphi)}{\partial \varphi} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

momentum conservation equations:

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_r}{\partial \varphi} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \varphi^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\varphi}{\partial \varphi}\right]$$

$$\frac{\partial u_\varphi}{\partial t} + u_r \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi} + u_z \frac{\partial u_\varphi}{\partial z} + \frac{u_r u_\varphi}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} + g_\varphi + \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \frac{\partial^2 u_\varphi}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \varphi} - \frac{u_\varphi}{r^2}\right]$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_z}{\partial \varphi} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \varphi^2} + \frac{\partial^2 u_z}{\partial z^2}\right]$$

stress tensor:

$$\tau_{rr} = -p + 2\eta \frac{\partial u_r}{\partial r}$$

$$\tau_{\varphi\varphi} = -p + 2\eta \left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}\right)$$

$$\tau_{zz} = -p + 2\eta \frac{\partial u_z}{\partial z}$$

$$\tau_{r\varphi} = \eta \left[r \frac{\partial}{\partial r}\left(\frac{u_\varphi}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \varphi}\right]$$

$$\tau_{\varphi z} = \eta \left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right)$$

$$\tau_{\varphi z} = \eta \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

In spherical coordinates

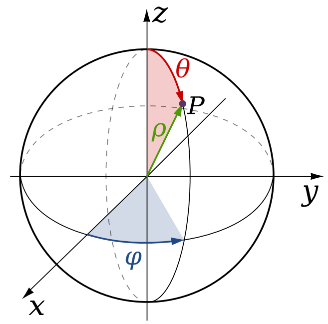

illustration: Wikimedia/Spherical_Coordinates

continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 u_r\right) + \frac{1}{r\sin(\theta)}\frac{\partial(\rho u_\varphi)}{\partial \varphi} + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\rho u_\theta\right) = 0$$

momentum conservation equations:

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\varphi}{r\sin(\theta)}\frac{\partial u_r}{\partial \varphi} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\varphi^2 + u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r +$$
$$\nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 u_r}{\partial \varphi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_r}{\partial \theta}\right) - \frac{2}{r^2}\left(u_r + \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta}{\tan(\theta)}\right) - \frac{2}{r^2\sin(\theta)}\frac{\partial u_\varphi}{\partial \varphi}\right]$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\varphi}{r\sin(\theta)}\frac{\partial u_\theta}{\partial \varphi} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta - u_\varphi^2\cot(\theta)}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta +$$
$$\nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 u_\theta}{\partial \varphi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_\theta}{\partial \theta}\right) + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{1}{r^2\sin^2(\theta)}\left(u_\theta + 2\cos(\theta)\frac{\partial u_\varphi}{\partial \varphi}\right)\right]$$

$$\frac{\partial u_\varphi}{\partial t} + u_r\frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r\sin(\theta)}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{r}\frac{\partial u_\varphi}{\partial \theta} + \frac{u_r u_\varphi + u_\varphi u_\theta\cot(\theta)}{r} = -\frac{1}{\rho r\sin(\theta)}\frac{\partial p}{\partial \varphi} + g_\varphi +$$
$$\nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\varphi}{\partial r}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_\varphi}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\left(2\sin(\theta)\frac{\partial u_r}{\partial \varphi} + 2\cos(\theta)\frac{\partial u_\theta}{\partial \varphi} - u_\varphi\right)\right]$$

stress tensor:

$$\tau_{rr} = -p + 2\eta\frac{\partial u_r}{\partial r}$$

$$\tau_{\varphi\varphi} = -p + 2\eta\left(\frac{1}{r\sin(\theta)}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{u_\theta\cot(\theta)}{r}\right)$$

$$\tau_{\theta\theta} = -p + 2\eta\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)$$

$$\tau_{r\theta} = \eta\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$

$$\tau_{\theta\varphi} = \eta\left(\frac{1}{r\sin(\theta)}\frac{\partial u_\theta}{\partial \varphi} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \theta} - \frac{u_\varphi\cot(\theta)}{r}\right)$$

$$\tau_{\varphi r} = \eta\left(\frac{\partial u_\varphi}{\partial r} + \frac{1}{r\sin(\theta)}\frac{\partial u_r}{\partial \varphi} - \frac{u_\theta}{r}\right)$$