Experimental Study of a Granular Gas Homogeneously Driven by Particle Rotations

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Abstract. We report an experimental study of a dilute "gas" of magnetic particles subjected to a vertical alternating magnetic field in a 3D container. Due to the torque exerted by the field on the magnetic moment of each particle, a spatially homogeneous and random forcing is reached where only rotational motions are driven. This forcing differs significantly from boundary-driven systems used in most previous experimental studies on non equilibrium dissipative granular gases. Here, no cluster formation occurs, and the equation of state displays strong analogy with the usual gas one apart from a geometric factor. These observations and the measurement of collision statistics at a container wall are well explained by a simple model, and enable to better understand out-of-equilibrium systems uniformly "heated".

Keywords: Granular gases ; Experiments ; Homogeneous forcing ; Equation of State
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INTRODUCTION

Granular gases display striking properties compare to molecular gases: cluster formation at high enough density [1–3], anomalous scaling of the pressure [2, 3] and of the collision frequency [4], non-Gaussian distribution of particle velocity [5]. These differences are mainly ascribed to dissipation occurring during inelastic collisions between particles. A continuous input of energy is thus required to reach a non equilibrium steady state for a granular gas. This is usually performed experimentally by vibrating a container wall or the whole container. For such vibration-fluidized systems, the role of the boundary condition affects the shape of the particle velocity distribution [5], as well as the extent of energy nonequipartition [6]. A spatially homogeneous forcing, driving each particles randomly, is thus needed to probe the validity domain of granular gas theories, but is hardly reachable experimentally [7]. Here, we experimentally report the equation of state and the collision statistics of a spatially homogeneous driven granular gas in a 3D container [8]. Magnetic particles subjected to a magnetic field oscillating in time are used to homogeneously and randomly drive the system by injecting rotational energy in each particle. Rotational motions are transferred in translational ones by the collisions with the boundaries or particles. To our knowledge, this type of forcing has been only used to investigate the pattern formation of magnetic particles either in a 2D cell or suspended on a liquid surface [9], as well as the velocity distribution of magnetic particles in a 2D cell [10]. Beyond direct interest in out-of-equilibrium statistical physics, granular medium physics, and geophysics (such as dust clouds or planetary rings [11]), our study provide insight into applied problems such as magnetic hyperthermia for medical therapy [12] or electromagnetic grinders in steel mills [13], both being based on control of magnetic particle dynamics by alternating magnetic field.

EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. A cylindrical glass container, 10 cm in diameter and 14 cm in height, is filled with N magnetic particles, with $2 \leq N \leq 60$ corresponding to less than 1 layer of particles at rest. Magnetic particles are made of a disc permanent magnet (5 mm in diameter and 2 mm thick) encased in a plexiglass cylinder (d=1 cm in outer diameter, 0.5 cm thick, and $L = 1$ cm long), both axes being collinear (see pictures in Fig. 1). This configuration enables a significant reduction of the dipole-dipole interaction between two particles. The magnetic induction of this dipolar particle, $\mu_0 \mathscr{M} = 250$ G is measured by a Hall probe at the top of the cylinder, $\mathscr{M}$ being the magnetization of the particle or its magnetic moment per unit volume, and $\mu_0 = 4\pi 10^{-7}$ H/m. The container is placed between two coaxial coils, 18 cm (40 cm) in inner (outer) diameter, 12.5 cm apart, the container and the coil axes being collinears as in Fig. 1. A 50 Hz ac current is supplied to the coils in series by a variable autotransformer (Variac 260V/20 A). An ac vertical magnetic induction B is thus generated in the range $0 \leq B \leq 225$ G with a frequency $f = 50$ Hz. Due to the Helmholtz configuration of the coils, B is homogeneous within the container volume with a 3% accuracy. Motions of particles are visualized with a fast camera with a 250 fps. An accelerometer stuck on the lid records the collision frequency and the

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impact amplitude on the lid during 500 s, the sampling frequency being 100 kHz to resolve collisions ($\sim 60\ \mu$s). We focus here on the dilute regime with volume fractions of $0.2\% \leq NV_p/V \leq 8\%$, with V the container volume and $V_p = \pi d^2 L/4 \simeq 0.78$ cm^3 the particle volume.

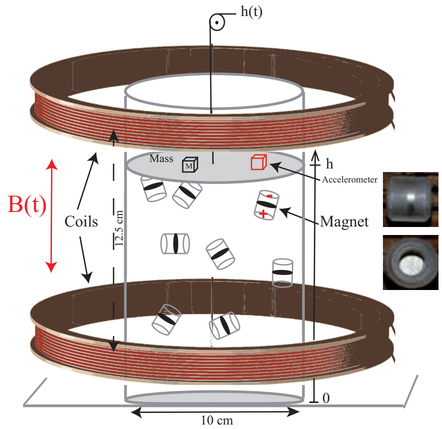

FIGURE 1. Experimental set up. Right insets shows pictures of a magnetic particle (1 cm scale).

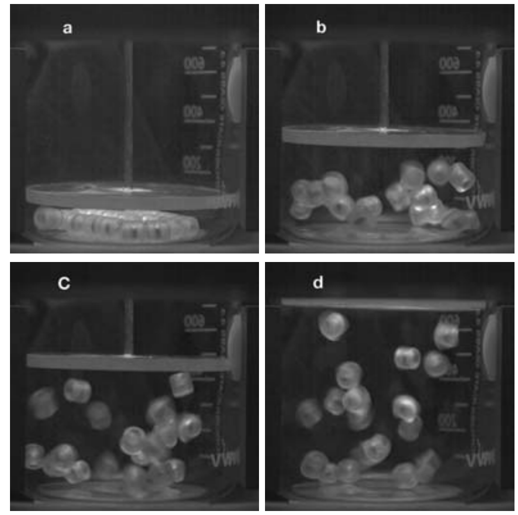

FIGURE 2. Snapshots of magnetic granular gas. $N = 20$. (a) Initial conditions: $B = 0$, a plexiglass lid is laying on the particles. When B is increased from (b) to (d), a gas-like regime develops and the lid rises up due to the particle collisions on it.

EXPERIMENTAL RESULTS

N particles are placed at the bottom of the container, their axes being in the horizontal plane, normal to B (see Fig. 2a). A plexiglass lid lays on the particles, its mass being balanced by a counterweight. When B is increased, a transition occurs at a critical B_c where particles begin to jump and lift up the lid. We found that $B_c = 75 \pm 5$ G whatever N. When B is further increased a stationary gas-like regime is observed with particles rotating and translating erratically - see Fig. 2b-d and movies in [14]. We observe that the rotation axis of most particles is normal to the particle axis in order to align the direction of its magnetic moment with the vertical oscillating magnetic field. The frequency and the direction of particle rotation is erratic, showing unpredictably reversing, and is thus not synchronized with the forcing frequency f.

Measurements are performed as follows. A mass M is added on the lid ($0.82 \leq M \leq 10$ g with a 0.82 step) and the lid is stabilized due to the particle collisions at a height that depends on B (constant-pressure experiment). The height $h(t)$ reached by the lid exhibits fluctuations in time around a mean height $\langle h \rangle$ - see inset of Fig. 3. $h(t)$ is measured by an angular position transducer at a 200 Hz sampling frequency during 200 s, the sensor output voltage being linear with the angle, and h. Note that the results reported here are unaffected when performing constant-volume experiments (the lid height is kept constant by adding a mass on the lid that depends on B).

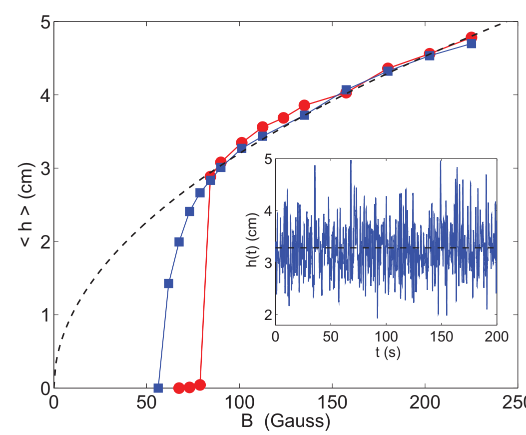

FIGURE 3. Hysteretic evolution of $\langle h \rangle$ for an increasing (•) and decreasing (■) magnetic field B. $N = 10$, $M = 4.7$ g. Dashed line is $\langle h \rangle \sim B^{1/2}$. Inset: Typical temporal evolution of $h(t)$ for $B = 101$ G, $N = 10$ and $M = 4.7$ g. Dashed line is $\langle h \rangle$=3.3 cm.

The mean height $\langle h \rangle$ reached by the lid is shown in Fig. 3 as a function of B for fixed N and M. The onset of the particle fluidization is hysteretic, occurring at B_c^i for increasing B, and at $B_c^d < B_c^i$ for decreasing B. One finds $B_c^d = 56 \pm 1$ G and $B_c^i = 75 \pm 5$ G whatever N. The thresholds come from the balance between magnetic energy, E_m, of a particle and the gravity energy, $E_g = mgd$, needed to vertically move it over its diameter, d, where $m = 1$ g is the particle mass, V_p its volume, and

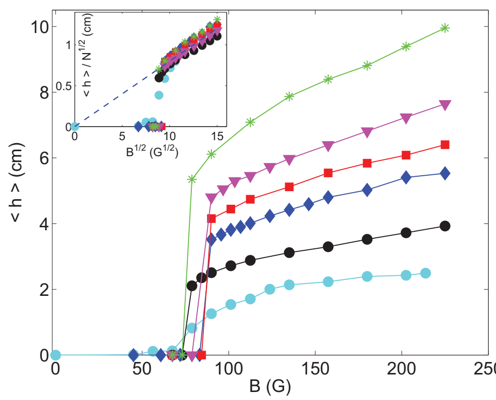

FIGURE 4. $\langle h \rangle$ vs. increasing B for different particle numbers $N = 4, 10, 15, 20, 30$ and 40 (from bottom to top). $M = 6.9$ g. Inset: best rescaling $\langle h \rangle / N^{1/2}$ vs. $B^{1/2}$.

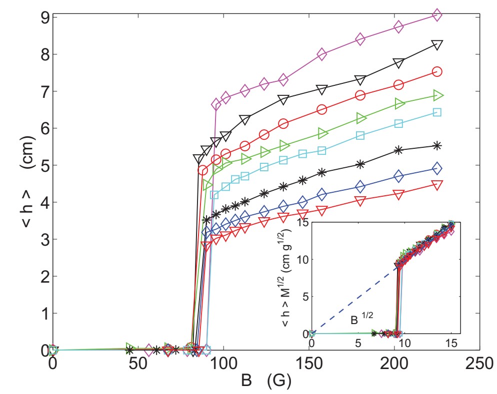

FIGURE 5. $\langle h \rangle$ vs. increasing B for different masses added $M = 2.3$, 3.1, 3.9, 4.7, 5.3, 6.9, 8.6 and 10 g (from top to bottom). $N = 15$. Inset: best rescaling $\langle h \rangle / M^{1/2}$ vs. $B^{1/2}$.

g the acceleration of gravity. When B is decreased, E_m corresponds to the particle dipolar energy, $E_d = \mathscr{M} V_p B$, and one finds $B_c^d = mgd/(\mathscr{M} V_p) \simeq 63$ G. When B is increased, particles are initially in contact, and E_m is the sum of the dipole-dipole interaction energy of two particles in contact, $E_{dd} = \mu_0 \mathscr{M}^2 V_p / 12$ [15], and the dipolar energy of a single particle E_d. By balancing $E_{dd} - E_d$ with E_g, one has $B_c^i - B_c^d = \mu_0 \mathscr{M} / 12 \simeq 21$ G as found experimentally. The hysteresis is thus due to the additional field needed to separate two particles initially in contact.

Far from the onset, Fig. 3 shows that $\langle h \rangle$ scales as $B^{1/2}$ meaning that the gaseous regime expands more and more when B increases. For fixed M, $\langle h \rangle$ is shown in Fig. 4 as a function of B for different particle numbers N. The larger N is, the higher is the height reached by the lid for a fixed B. The best rescaling is displayed in the inset of Fig. 4, and shows that $\langle h \rangle / N^{1/2} \sim B^{1/2}$. For fixed N, $\langle h \rangle$ is shown in Fig. 5 as a function of B for different added mass M on the lid. The larger M is, the smaller is the height reached by the lid for a fixed B. The best rescaling is displayed in the inset of Fig. 5, and shows that $\langle h \rangle M^{1/2} \sim B^{1/2}$. To sum up, one finds an experimental state equation for the magnetic granular gas

$$M \langle h \rangle^2 \sim NB \,. \qquad (1)$$

MODEL

Assume θ the angle between the vertical field B and a magnetic moment of a particle. A torque $\mathscr{M} V_p \times B$ is thus exerted by the field on the magnetic moment of each particle. The kinetic momentum theorem writes $I d^2\theta(t)/dt^2 = \mathscr{M} V_p B \sin \omega t \sin\theta$, with $I = m(3d^2/4 + L^2)/12 = 1.4\ 10^{-8}$ kg.m^2 the moment of inertia of the particle. This equation is known to display periodic motions, period doubling, and chaotic motions [16]. The ratio between the magnetic energy, E_d, and the rotation energy, $E_{rot} = I\omega^2/2$ controls the stochasticity degree. The synchronization between the angular velocity of the particles and the magnetic field one, $\omega = 2\pi f$, is predicted to occur when $E_d \ll E_{rot}$, that is $B \ll I\omega^2/(2\mathscr{M} V_p) = 220$ G. When this condition is violated, as it is the case for our range of B, chaotic motions occur [16]. The external magnetic field thus generates an erratic rotational driving of each particles. A spatially homogeneous forcing is thus obtained where only the rotational degrees of freedom of each particle are erratically driven.

Particle-particle and particle-boundary collisions have a key role. They "randomize" the angular momentum of the particles, they dissipate their energy, and this is the only way to give a translational velocity, v, to the particles. Assume that a loss of kinetic energy during a collision is compensated by a gain magnetic energy of the particle. Then, one has $\mathscr{M} V_p B(t) \sim v^2$. Thus, the typical particle velocity scales as

$$v(B) \sim \sqrt{B} \,. \qquad (2)$$

More precisely, from the inelastic collision rules, the energy loss by a particle of mass m during a collision with the lid of mass M writes $mMv^2(1-\varepsilon^2)/[2(m+M)]$, ε being the particle-boundary restitution coefficient. The energy balance finally leads to $v = \sqrt{\mathscr{M} V_p B (m+M)/[mM(1-\varepsilon^2)]}$.

On the other hand, the time of flight τ of a single particle between 2 collisions with the lid (at the altitude h) reads simply $\tau = 2h/v$ (neglecting gravity). For N particles, this time is assumed to be divided by the volume fraction NV_p/V, with $V = Sh$, S being the container area, and thus

$$\tau = \frac{2h^2 S}{N v(B) V_p} \ . \quad (3)$$

The time of flight under gravity of the lid is $\tau_l = v_l/g$ with v_l the lid velocity due to the particle collisions on the lid. When both times are synchronized $\tau_l = \tau$, one has $h^2 = N v v_l V_p/(2gS)$. From the inelastic collision rules, the lid velocity $v_l \sim v$, and writes $v_l = vm(1+\varepsilon)/(m+M)$. The gas expansion thus reads $h \sim N^{1/2} v$. Substituting Eq. (2) into this equation thus leads to the theoretical state equation

$$Mh^2 \sim NB \ , \quad (4)$$

in good agreement with the experimental one of Eq. (1).

DISCUSSION

We have obtained the equation of state of a dissipative granular gas driven homogeneously by the rotations. With usual notations (the pressure P on the lid $\sim Mg/S$, and the container volume $V = Sh$), the equation of state thus reads $PV \sim NE_c \frac{V_p}{V}$, with $E_c \sim \langle v^2 \rangle \sim B$ the mean kinetic energy of particles of velocity v. Surprisingly, this equation is close to the equation of state of a perfect gas ($PV = NE_c$) with a geometric correction: the particle-container volume ratio. Moreover, it differs from the equation of state of a dissipative granular gas driven by a vibrating wall $PV \sim E_c$ with $E_c \sim \mathscr{V}^{\theta(N)}$ with $\mathscr{V}$ the forcing velocity of the wall, and $\theta(N)$ a decreasing function from $\theta = 2$ at low N to $\theta \simeq 0$ at large N when the clustering phenomenon occurs [3]. Here, no clustering is observed even when the volume fraction is increased up to 40%. We also show that the magnetic field B in our experiment is the analogous of the thermodynamic temperature for molecular gases, or the analogous of the granular temperature for dissipative granular gases since one has $\langle v^2 \rangle \sim B$. Finally, the collision frequency $\sim 1/\langle \tau \rangle$ is found to scale as $N\sqrt{B}$ (not shown here). This result is consistent with the frequency collision of a perfect gas $\sim N\sqrt{\langle v^2 \rangle}$, but not with the one of a vibro-fluidized dissipative granular gas in a dilute regime $\sim N^{1/2}\mathscr{V}$ [4]. This difference is related to the spatially homogeneous forcing.

CONCLUSION

We have experimentally studied, for the first time, a 3D granular gas driven homogeneously in volume by particle rotations. This differs from previous experimental studies of granular gas where the energy was injected by vibration at a boundary. A state of equation is experimentally identified and the scaling of the mean frequency collision with the forcing obtained. Several differences are reported with respect to thermodynamiclike gas or non equilibrium vibro-fluidized dissipative granular gas: (i) the gas-like state equation has a geometric correction (container-particle aspect ratio), and (ii) no cluster formation occurs at high density. The use of this new type of forcing will be of primary interest to experimentally probe the distribution of particle velocity [8], and to test the possible equipartition of rotational and translational energy, a feature not guaranteed for out-of-equilibrium systems [17].

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