Threshold of gas-like to clustering transition in driven granular media in low-gravity environment

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Supplementary material

The aim of this supplementary material is to derive Eq. (2) from Eq. (1) of the main text.

By assuming a uniform distribution of the grains in the slice, the probability of presence of one grain in a specific column is $p = d/l$. If the slice contains N_y particles, the probability $P(X = n)$ of finding n grains in the column is given by the binomial distribution

$$P(X = n) = \binom{N_y}{n} p^n \, (1-p)^{N_y - n} \,, \tag{1}$$

where X is the random variable representing the number of grains in a column, and $\binom{N_y}{n} = N_y!/[n!(N_y - n)!]$ the binomial coefficient. Remembering that the number of particles n in each column being actually unknown, the number of particles in the slice is a priori unknown too. However, one might observe the realization of the event $X = 0$. Effectively, if there is no grain in a column, the light will go through the system and reach the camera. A white zone is then detected in the corresponding square. Using the previous equation, we deduce the probability to observe a white zone as

$$P(X = 0) = (1-p)^{N_y} \,.$$

In order to deduce the relationship between the observable shadow density s_x and the likely number of particles from which it comes N_y, we introduce the random variable O_x which quantifies the shadow, casted by the particles in the related column. This random variable is defined as

$$O_x = \begin{cases} 0 \text{ if } X = 0 \\ 1 \text{ if } X \geq 1 \end{cases} ,$$

with the associated probabilities $P(O_x = 0) = P(X = 0)$ and $P(O_x = 1) = 1 - (1-p)^{N_y}$. The expected value of the random variable O_x is given by $E[O_x] = 1 - (1-p)^{N_y}$. To estimate this expected value, we introduce finally the observable o_x as the number of black pixels in the square divided by the surface of the square expressed in pixels. The average of this observable is not only the estimator of the expected value but also the measured shadow density, $s_x = \langle o_x \rangle = E[O_x]$, and one finds finally

$$s_x = 1 - (1 - p)^{N_y} \, .$$

By isolating N_y in this last relation, we found the most probable number of particles in the slice. One obtains

$$N_y = \frac{\ln(1 - s_x)}{\ln(1 - d/l)} \cdot \tag{2}$$