Experimental determination of a state equation for dissipative granular gases

Détermination expérimentale d'une équation d'état pour les gaz granulaires dissipatifs

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Running title: State equation for dissipative granular gases

RÉSUMÉ

Nous utilisons des mesures de pression et de volume d'un gaz de particules sphériques excitées par un piston vibrant et interagissant par des collisions inélastiques, pour déterminer une équation d'état reliant pression, volume, nombre de particules, et amplitude et fréquence de vibration.

Mots-clé : gaz granulaire, équation d'état.

ABSTRACT

From measurements of pressure and volume of a gas of spherical particles excited by a vibrating piston and undergoing inelastic collisions, we determine a state equation between pressure, volume, particle number and the vibration amplitude and frequency.

Keywords: : granular gas, state equation.

INTRODUCTION

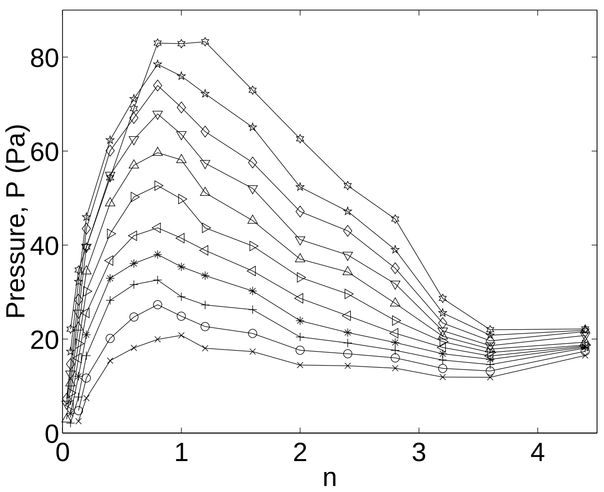
We report an experimental study of a "gas" of inelastically colliding particles, excited by vertical vibrations. When the vibration is strong enough and the number of particles is low enough, the particles display ballistic motion between successive collisions like molecules in a gas [1]. The aim of this study is to try an experimental determination of the state equation of this dissipative gas. It is known that for a fixed number n of granular layers at rest, one has in the low density limit $P\Omega \propto T$ [2], where P is the mean pressure, Ω the volume and T the "granular temperature", *i.e.* the mean kinetic energy per particle. However, the dependence of T on the vibration amplitude, A, and frequency, f, of the piston and on the number of particles is still a matter of debate [3-6]. Kinetic theory [6, 7] or hydrodynamic models [4] show $T \propto V^2 n^{-1}$, whereas numerical simulations [8-11] or experiments [7, 8] give $T \propto V^{\alpha} n^{-\beta}$, with $1.3 \leq \alpha \leq 2$ and $0.3 \leq \beta \leq 1$, where $V=2\pi fA$ is the maximum velocity of the piston. Our previous density measurements [1] showed an exponential "atmosphere" far enough the piston from which we extracted a granular temperature V-dependence of the form $T \propto V^{\theta(n)}$, with θ continuously varying from $\theta = 2$ when $n \rightarrow 0$, as expected from kinetic theory, to $\theta \approx 0$ for large n.

EXPERIMENTAL RESULTS

The experiment consists of a transparent cylindrical tube, *60* mm in inner diameter, filled from *20* to *2640* stainless steel spheres, *2* mm in diameter. *600* particles correspond to $n = 1$ particle layer at rest. An electrical motor, with eccentric transformer from rotational to translational motion, drives the particles sinusoidally with a *25* mm amplitude, A, in the frequency range $9 \leq f \leq 20$ Hz. A lid in the upper part of the cylinder, is either fixed at a given height, h (constant-volume experiment) or is stabilized at a given height h_m due to the bead collisions (constant-pressure experiment). Heights h and h_m are defined from the lower piston at full stroke.

Time averaged pressure measurements have been done as follows. Initially, a counterwheight of mass *46* g balances the lid mass. The piston drives stainless steel spheres in erratic motions in all directions [1]. Particles are hitting the lid all the time, so that to keep it at a given height h, we have to hold the lid down by a given force, Mg, where M is the mass of a weight we place on the lid and g the acceleration of gravity. At a fixed *h*, *i.e.* at a constant-volume, Fig. 1 shows the time averaged pressure $P=Mg/S$ exerted on the lid as a function of the number *n* of layers at rest in the container, for different frequencies of vibration, *S* being the area of the tube cross-section. At constant external driving, *i.e.* at fixed *f* and *A*, the pressure passes through a maximum for *0.8* particle layers at rest. This critical number is independent on the vibration frequency. A futher increase of the number of particles leads to a decrease in the mean pressure underlying that more and more energy is dissipated by inelastic collisions. Note that gravity has a small effect in these measurements that are performed for $V^2 \gg gh$. For $n < 1$, most particles are in vertical ballistic motion between the piston and the lid. Thus, the mean pressure increases roughly proportionally to *n*. When *n* is increased such that one has more than one particle layer at rest, interparticle collisions become more frequent. The energy dissipation is increased and thus the pressure decreases.

We now consider the bed expansion under the influence of collisions on a circular wire mesh lid placed on top of the beads leaving a clearance of about *0.5* mm between the edge of the lid and the tube one. Due to the bead collisions, the lid is stabilized at a given height h_m from the piston at full stroke. Although the lid mass is roughly *50* times smaller than the total mass of beads, the lid proves to be quite stable and remains horizontal. The expansion, $h_m - h_0$, of the bed is displayed in Fig. 2 as a function of *n* for different vibration frequencies. h_0 is the bed height at rest. At fixed *f*, the expansion passes through a maximum for *0.6* particle layers at rest. This critical number is independent on the vibration frequency. When *n* is further increased, the expansion decreases showing, as for pressure measurements, an increase in dissipated energy by inelastic collisions. Note that the height h_m of the granular gas is much larger than for pressure measurements of Fig. 1. Consequently, gravity is obviously important. Moreover, for a given *n*, the number of interparticle collisions is larger than for the pressure measurements.



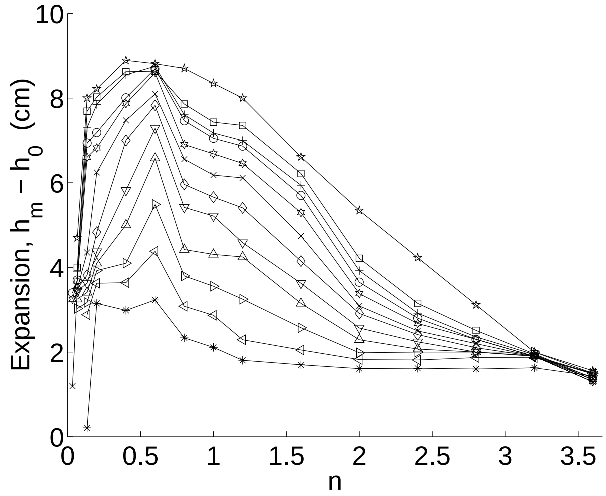
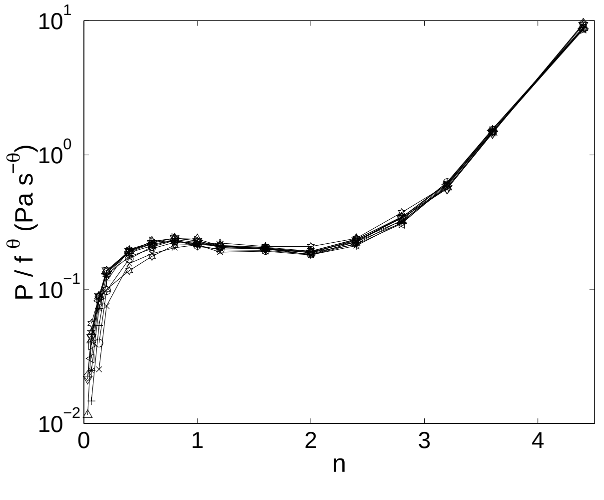


Figure 1 : Mean pressure P as a function of n. From the upper (×) to the lower (hexagrams) curve, vibration frequency f varies from 10 to 20 Hz with a 1 Hz step. For all these experiments, $h - h_0 = 5$ mm. Lines join the data points.

Figure 2 : Maximal bed expansion, h_m- h_0, as a function of n. From the lower (✳) to the upper (☆) curve, vibration frequency f varies from 9 to 20 Hz with a 1 Hz step. Lines join the data points.



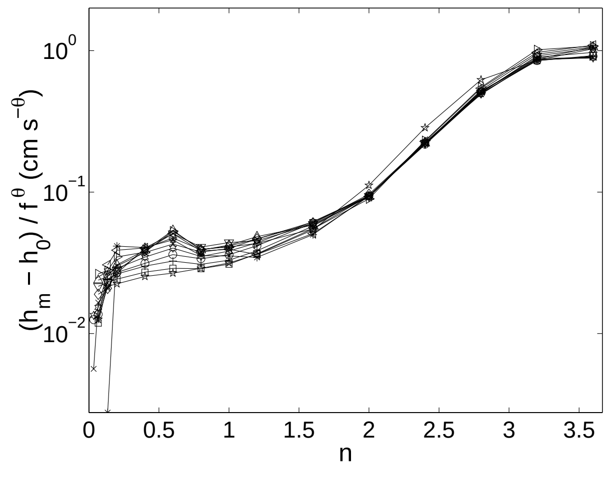


Figure 3 : Mean pressure P from Fig. 1 rescaled by f^{θ} as a function of n. $\theta(n) = 1 - \tanh(n-n_c)$ with $n_c = 3.5$.

Figure 4 : Maximal bed expansion, h_m- h_0, from Fig. 2 rescaled by f^{θ} as a function of n. $\theta(n) = 1 - \tanh(n-n_c)$ with $n_c = 2.3$.

A STATE EQUATION

In order to use the above measurements to try to determine a state equation, we have to find the appropriate dependence $T = T(V, n)$ of the granular temperature as a function of the vibrating velocity V and the number of particle layers n. Taking into account the law $P\Omega \propto T$ for small densities and our previous observation $T \propto V^{\theta(n)}$ from density measurements in an exponential atmosphere, we have plotted $Log\ P$ and $Log\ (h_m - h_0)$ as functions of $Log\ V$. On the reported frequency range, these curves are straight lines, the slopes of which give $\theta(n)$. The behavior of $\theta(n)$ for the experiments at constant volume (resp. constant pressure) is displayed in Fig. 5 with □-mark (resp. ◊-mark) together with the one in ○-mark extracted from exponential density profiles of Ref. [1]. The three curves, obtained with different experimental conditions and independent measurement have the same shape which could be simply fitted by $\theta = 1 - tanh(n - n_c)$ where $n_c = 3.5$ (resp. 2.3) and 2.

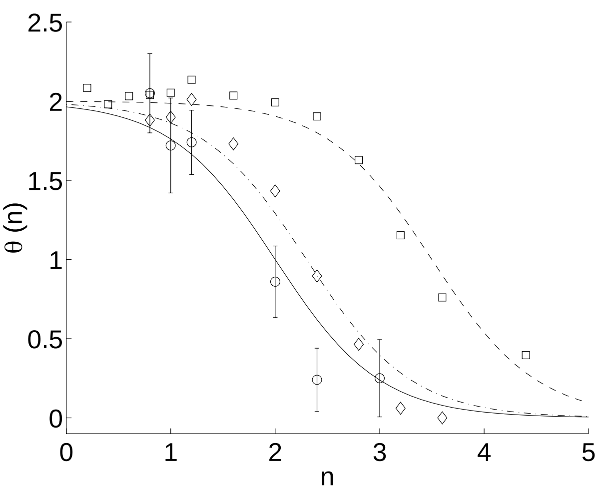


Figure 5 : Evolution of the θ exponent as a function of the number of layers, n, from pressure (□), bed expansion (◊) and density (○) measurements. Fits are $\theta(n) = 1 - tanh(n-n_c)$ with $n_c = 2$ (—), 2.3 (— •) and 3.5 (——).

We can now use the observed law $T \propto V^{\theta(n)}$ to scale the pressure and bed expansion measurements of Fig. 1 & 2. The results are displayed in Fig. 3 & 4 and show a rather good collapse of all the data on a single curve. We have thus shown that the law, $P\Omega \propto T$, together with $T \propto V^{\theta(n)}$, provide a correct empirical state equation for our dissipative granular gas in the kinetic regime. As shown earlier, this regime is limited at high density by the clustering instability [1, 12, 13] and on the other side, for a fixed too small number of particles, when the gas suddenly contracts on the piston below a critical frequency [1, 8, 14, 15].

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