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Microgravity experiments on vibrated granular gases in a dilute regime: non-classical statistics

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Abstract. We report on an experimental study of a dilute gas of steel spheres colliding inelastically and excited by a piston performing sinusoidal vibration, in low gravity. Using improved experimental apparatus, here we present some results concerning the collision statistics of particles on a wall of the container. We also propose a simple model where the non-classical statistics obtained from our data are attributed to the boundary condition playing the role of a 'velostat' instead of a thermostat. The significant differences from the kinetic theory of usual gas are related to the inelasticity of collisions.

Keywords: finite-size scaling, coarsening processes (experiment), stochastic particle dynamics (experiment), granular matter

Contents

1. Introduction

A tremendous number of works have been devoted to the study of granular gas within the past fifteen years [1]–[3]. But a series of basic questions remain to be addressed: what is the effect of particle rotations [4]? Does the vibrated container play the role of a thermostat or of a velostat [5]? Is the limiting case of particles interacting only with the walls the same as that of particles interacting with each other? What is the true role of the boundaries and of the gravity? These last three points were previously investigated in [6] and demonstrated the importance of rotation and boundary conditions for very dilute systems.

However, many works devoted to granular gas are often addressed from a theoretical point of view without gravity or particle rotation, or by means of two-dimensional (2D) numerical simulations without taking rotation into account [7, 8]. In the same way, most of the experimental results are obtained in 2D experiments in the presence of gravity [9]. It is then difficult to achieve a complete comparison in such conditions. This highlights the lack of true 3D experiments in microgravity to test the theoretical and numerical results.

The present paper reports on 3D experiments of a dilute granular medium strongly fluidized by sinusoidal vibrations in a low gravity environment and where impacts of particles are recorded by a force sensor. This paper is a continuation of previous experiments [10] where non-classical statistics were found. The force sensor has been calibrated showing that the maximum amplitude, I, of detected impact scales as the maximum force in a Hertz contact leading to $I \propto v^{6/5}$, where v is the particle's velocity at impact.

2. Experimental apparatus

As shown in figure 1(a), the experimental set-up consists of a cylindrical cell ($D = 13$ mm, $H = 10$ mm at rest) closed at the bottom by an electromagnetically driven mobile piston

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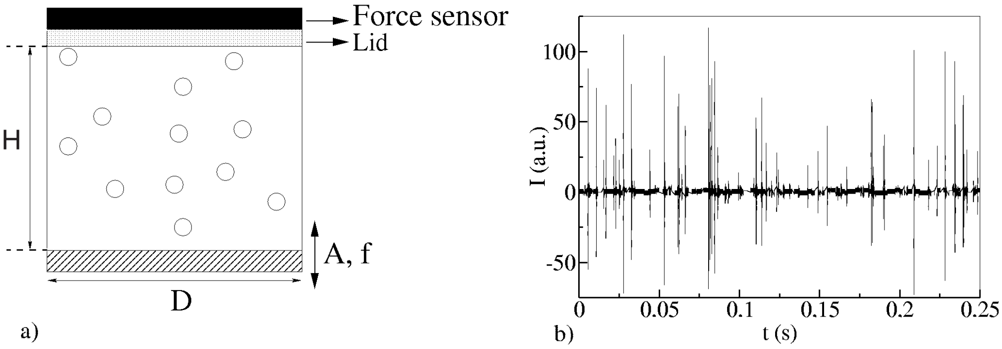

Figure 1. Experimental apparatus (a), and typical signal of amplitude I (b) recorded with the force sensor during 0.25 s (10 periods) of vibration. The parameters of vibration are $f = \omega/2\pi = 40$ Hz, $A = 1.11$ mm. The number of balls in the cell is: $N = 12$. The radius of the balls is $r = 1.00$ mm and the dimensions of the cell are $D = 13$ mm, $H = 10$ mm.

which vibrates according to $z = A\cos(\omega t)$ (A is the amplitude of vibration, $2A < 5$ mm, and $f = \omega/2\pi$ is the frequency of vibration, 30 Hz $< f <$ 120 Hz, $A\omega^2 < 480$ m s^{-2}) and at the top by a fixed force sensor working in the impacting regime [6, 10]. The cell contains N steel spheres of radius $r = 1$ mm ($N = 12$, 24, 36, 48, corresponding respectively to $n = 0.3$, 0.6, 0.9, 1.2 layers at rest). The apparatus is placed onboard the Airbus A300-0g of CNES which delivers $0.0g \pm 0.05g$ during a series of parabolas lasting 20 s each. Figure 1(b) shows a typical recording of the signal from the force sensor. It exhibits a series of peaks of maximum amplitude I, known as impact hereafter. Compared to [6, 10, 11], the gauge has now been covered by a lid of given thickness to avoid non-uniform response of the sensor. Moreover, calibration of the force covered sensor using impacts from 1-ball experiments [6] have shown that the maximum amplitude of each peak behaves as the maximum force in a Hertzian contact.

The experiment consists of studying the collision frequency and amplitude as functions of the number of balls in the cell and of the typical piston speed $V = A\omega$.

3. Experimental results

In order to make the comparison with [10] easier, the same protocol is applied: the collision statistics are obtained from the 16 central seconds of the 20 s of zero gravity condition—to avoid transients—and the data are presented with the same axis.

Figure 2(a) gives an example of the number N_c of collisions with the sensor divided by the duration of the measurement (16 s), for a cell containing 12 balls at different piston speeds $V = A\omega$. The linear behaviour shows that the controlling parameter is the piston speed V. Moreover, as shown in figure 2(b), this trend is independent of the number of balls in the cell, within the investigated range of $A \in [0.79–1.21$ mm$]$, $f \in [40–121$ Hz$]$ and $N \in [12–48]$. So the average speed of the balls in the direction of vibration $\langle |v_{b_z}| \rangle$ scales linearly with V.

Figure 2(b) strengthens the result of [10]. It shows with different and less noisy data that the number of collisions N_c scales linearly with $N^{0.6}$ within the experimental range

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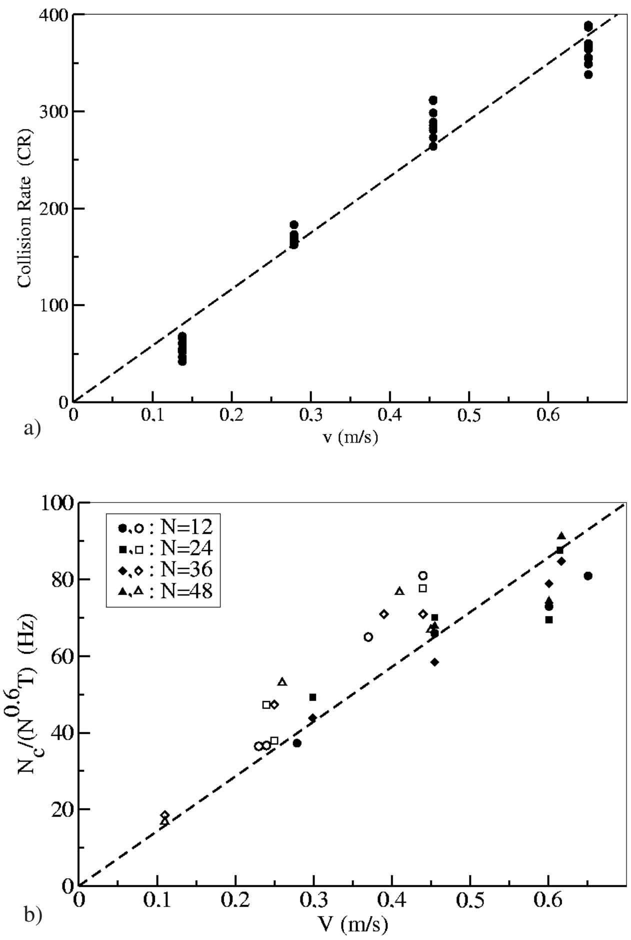

Figure 2. (a) Typical variation of the number of collisions per second, averaged over different periods of time (1, 2.5 and 5 s) and at different instants (after 2, 5 and 10 s), for 12 balls, as a function of the piston speed $V = A\omega$. One then has nine values for each V giving an estimation of the typical deviation to the mean. The dashed line is a guide for the eye. (b) Total number of collisions N_c observed during time $T = 16$ s of low gravity, rescaled by $N^{0.6}$: $N_c/(TN^{0.6})$ as a function of V for $N = 12, 24, 36$ and 48. The empty symbols are from a previous set of experiments [10]. N is the number of particles in the cell. The dashed line corresponds to the fit $N_c/(TN^{0.6}) = \alpha V$.

of investigated parameters, $N_c \propto N^{0.6}$. Note that if the dynamics of the particles were not correlated, N_c should scale as N. The latter observation allows determination of some typical average ball speeds $\langle|v_{b_z}|\rangle$ in the direction of vibration. Let us define $\langle|v_{b_z}|\rangle$ as the mean particle velocity in the direction of vibration. The total distance travelled by all the particles in this direction during the time T is $N\langle|v_{b_z}|\rangle T$ and the corresponding number of impacts with the sensor gives $N_c = N\langle|v_{b_z}|\rangle T/2(H - 2r)$. One deduces (with $N_c \propto VN^{0.6}$): $\langle|v_{b_z}|\rangle \propto (V/N^{0.4})(H - 2r)$.

The typical impact recording (figure 1(b)) gives access to different statistics. In the following we study the distribution of waiting times between successive impacts and the distribution of impact maxima.

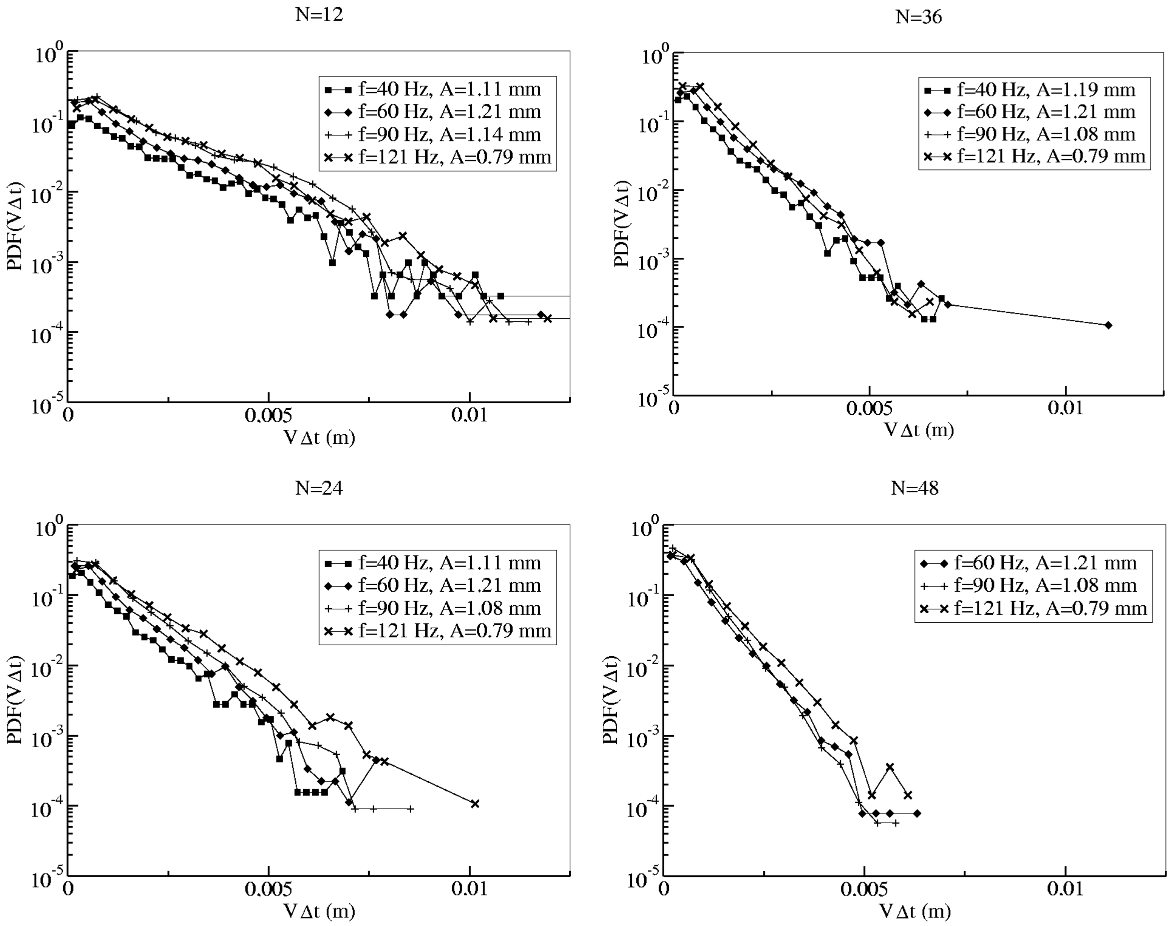

Figure 3. Probability density functions of the free time Δt separating two successive collisions detected by the impact sensor rescaled by $V = A\omega$, for different vibration parameters during 16 s of low gravity.

Figure 3 shows the probability density function (PDF) of the time lag Δt between two successive impacts with the sensor for several numbers of particles N and figure 4 shows the rescaled PDF leading to a single exponential curve $\approx \exp \frac{3}{2} N^{0.6} V \Delta t$. Figure 5 shows the PDF of the maximum of impact on the force sensor. Both distributions present exponential trends in agreement with those obtained in previous experiments with a less improved experimental apparatus [10]. If the number of balls N is less than 36, the PDF of the impact amplitude is not exponential (also observed in [10] for $N = 12$). This can be attributed either to an insufficient number of impacts, or to the saturation of the ball speed $v_{b_z} = 2(H - 2r)f$, leading to a superposition of two behaviours: the first one, at low I/V, when the ball motion is not synchronized with the piston and the second one, at larger I/V, corresponding to a progressive synchronization with the piston motion as in [6]. However, more experiments should be performed to validate this last idea.

The above results combined with previous ones [10] show the robustness of the exponential distributions for the PDF of Δt and I in this range of experimental parameters and conditions. However, we shall note that other kinds of distributions have also

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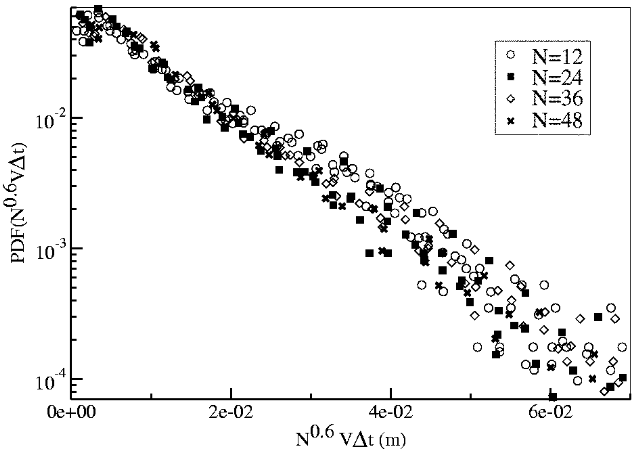

Figure 4. Probability density function of the free time Δt separating two successive collisions, rescaled by V ($V = A\omega$) and by the number of beads N.

been observed experimentally [6, 9, 12] with different experimental conditions—2D, 3D, gravity—highlighting the sensitivity of the problem.

It is then worth noting that none of the previous simulations or theoretical models find this kind of exponential distribution [14, 15], except with very special boundary rules [13]. Note also that the scaling of the number of collisions, $N_c \approx \sqrt{N}$, has recently been recovered in 2D simulations [16], but no information about the shape of the impact distribution was given and particle rotation has not been introduced, although it has recently been shown [4, 6] that rotation might be important in the 3D granular gas pattern of the 1-g oscillon type, and of the ultimate case of 0-g and very low density respectively.

In the next section, we present an interpretation of these exponential distributions found here.

4. Discussion

4.1. Time lag distribution as a Poisson process

From the exponential trend of the PDF of the time lag between two successive impacts, the impact series on the force sensor can be seen as a Poisson process on which one can perform several statistical tests. We have already shown that correlations might exist due to the anomalous scaling of the number of impacts with N. We want to confirm the importance of correlation and to show whether the Poisson process is homogeneous or not, that is, if the impact series corresponds to a stationary state.

We first perform a Kolmogorov–Smirnov test [17] of hypothesis H_0: 'the PDF of time lag is exponential with no correlation'. The test shows that H_0 is not achieved. We then fit our experimental PDF with an exponential law and find that the error is small, allowing us to conclude that the PDF is exponential but with correlation between events as expected.

The second test is a homogeneity test of the Poisson process. We plot the number of impacts between $t = 0$ and $t = t'$ for increasing t'. A likelihood ratio test shows that the number of impacts does not increase linearly with t' proving that the Poisson

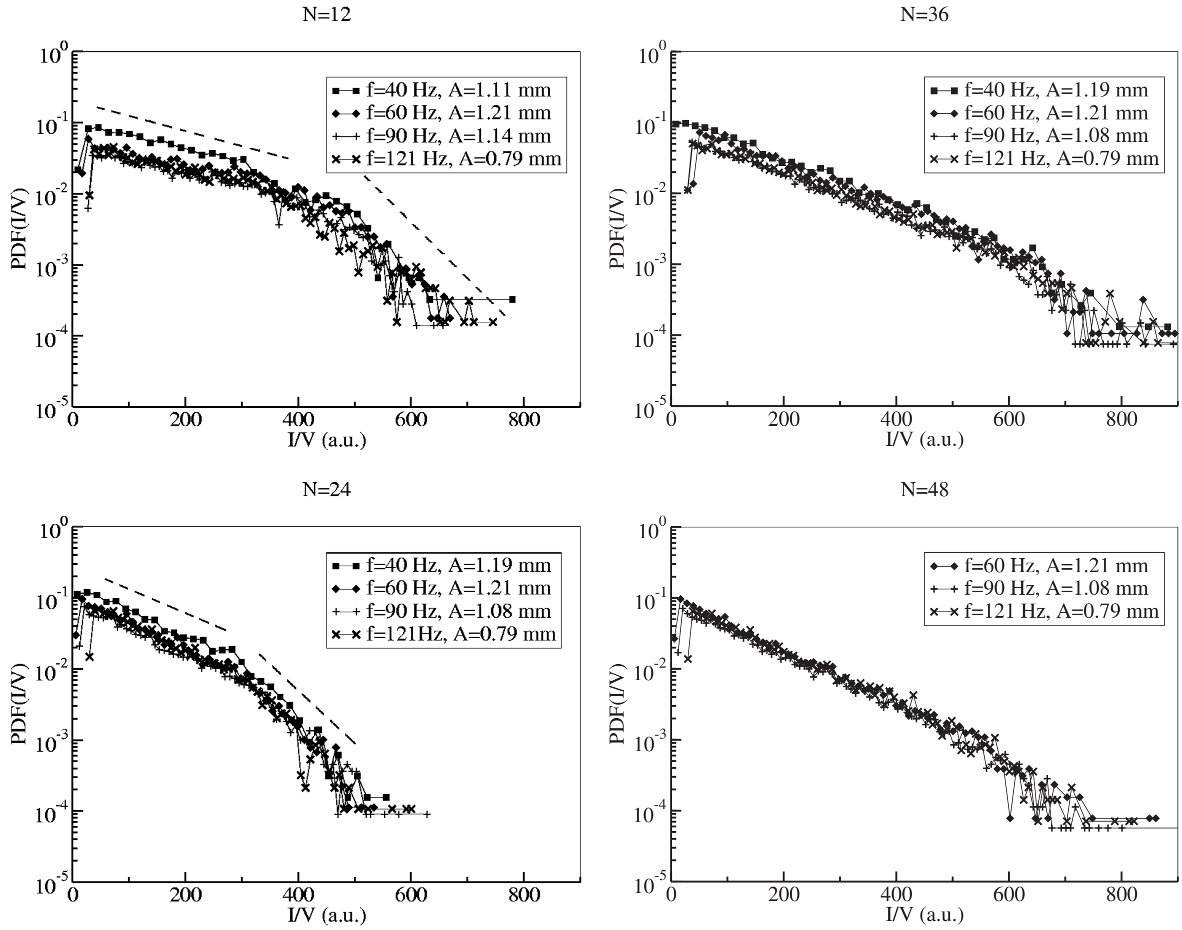

Figure 5. Probability density functions of the impact amplitude I measured by the sensor, for different vibration parameters during 16 s of low gravity, for different number of balls: $N = 12$, 24, 36 and 48.

process is non-homogeneous and consequently that the impact series do not correspond to a stationary state despite a large number of impacts per particle.

It seems that this non-stationarity cannot be attributed to the g-jitter because it would involve a variation of the speed of a particle much smaller than the minimum speed of the piston. In another context, Moon *et al* [4] have noticed that particles rotation plays an important role, not only by increasing the dissipation but also by modifying the coupling and increasing the real distribution of grains which can be observed. This might also be the case here, in particular including that the rotation degrees of freedom increases the time for a stationary state to be reached. It also points out the necessity to prove that the stationary state still exists in this experiment when rotations are considered. Longer experiments or numerical simulations with rotation and suitable statistical tests should be performed to address this issue.

4.2. A simple model 'à la Boltzmann' for the impact distribution

Recently, Van Zon *et al* [12, 18] have shown numerically that the velocity distribution depends on the boundary condition, while Campbell [19] casts a doubt on the relevance of kinetics theory to describe rapid granular flow particularly because of large dissipation. Here, we propose a model which takes into account the specific boundary condition and which is based neither on energy conservation nor on the importance of granular temperature.

Figure 5 shows that the ball speed at impact (i.e. close to the wall container opposite the piston) obeys approximately an exponential distribution, $\mathrm{PDF}(v_{b_z}) \approx \mathrm{e}^{-v_{b_z}/v_0}$. This may be explained by a simple model, first reported in [11]. Starting from the general framework of Boltzmann's equation, and from its stationary solution, let us consider that a statistical distribution can be written as a function of the collision invariants, that is here the total momentum of the particles rather than the total energy, because impacts between particle are inelastic. One might then consider that the quantity whose disorder has to be maximized is the momentum of the particles. This allows us to proceed as in classic statistical physics: one assumes that the observed state is the most probable one; so, it is the state with the largest possible complexion number W: W should be optimum, and $\ln(W)$ too. However, it should also correspond to a possible state of the system. Let us also assume that the mean speed is imposed by the coupling between the particles and an external system, known as a 'velostat'. Let n_i be the number of particles with speed v_i, the number of complexion W is: $W = N!/(n_1! n_i! \ldots)$. The two conservation laws write: $N = \sum_i n_i$, $N\langle v_\mathrm{b}\rangle = \sum_i v_i n_i$. Using $\mathrm{d}v_i = 0$, $\mathrm{d}W = 0$, $\mathrm{d}N = 0$ and the Stirling relation $\ln(N!) \approx N\ln(N) - N$, one gets: $\mathrm{d}\ln(W) = 0 = -\sum_i \ln(n_i)\,\mathrm{d}n_i$, $0 = -\sum_i \mathrm{d}n_i$, $0 = -\sum_i v_i \mathrm{d}n_i$. A way to condense these three equations into a single one is to use the Lagrange multiplier technique and then:

$$0 = -\sum_i [\ln(n_i) + \lambda_1 + \lambda_2 v_i]\,\mathrm{d}n_i \tag{1}$$

where λ_1 and λ_2 are fixed by the experimental conditions and the n_i are uncoupled to each other. With the normalization condition $\sum_i n_i \mathrm{d}v_i = N$, one has the general exponential solution:

$$\frac{n_i}{N}(v_i) = \lambda_2 \mathrm{e}^{-\lambda_2 v_i} \tag{2}$$

which is in agreement with figure 5 for $N = 36$ and 48. As a consequence, one can see the system of grains as coupled to a velostat—the moving piston—confirming recent assumptions [5, 11]. However, the typical speed of the balls depends on the mean free path compared to the cell size, that is the number of particles N. It means that the moving piston is not a true velostat, since it imposes a typical speed which depends on the number of balls and that the system does not follow an 'extensive' physics.

5. Conclusion

This experiment has investigated the statistics of collisions in a granular gas excited by a vibrating piston in micro-gravity conditions in the dilute regime. It follows up our previous observed experiment [10]. The regime studied here is intermediate between the very low density regime of non-interacting particles [6], and the high density one where clustering of particles probably occurs [20, 21].

It has been found here that the typical ball speed $\langle v_\mathrm{b}\rangle$ is proportional to the piston speed V, and decreases rapidly with N as $V/N^{0.4}$. The distribution of time lag between

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two successive collisions with a wall container is found to be exponentially decreasing with Δt. It has been interpreted as a non-homogeneous Poisson process. For large enough N, the experimental distribution of impacts amplitude I is found to decrease exponentially as I/V. An interpretation for this exponential tail is proposed: the piston does not play the role of a thermostat, but that of a 'velostat' or of a random impact generator. More generally the notion of temperature should also be taken with caution in the case of rapid granular flow [19].

Finally, in most experiments on dilute granular gas, the piston speed V goes faster than the typical ball speed, which indicates a supersonic excitation since the typical sound speed c of a normal gas is also its mean speed $\langle v_\mathrm{b} \rangle \approx c$ (within a $\sqrt{\gamma}$ factor, $\gamma = C_p/C_v$). Even if this continuous mechanics view point is not obvious at all in the Knudsen regime, it confirms previous analyses and findings on denser samples [21, 22]. These results cast a doubt on using continuous equations to model granular gases, because they demonstrate the difficulty of defining correct boundary conditions, correct local averages and correct coupling due to dissipation and/or the Knudsen regime. In particular, the role of dissipation has to be elucidated.

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