Supplemental Material to "From modulational instability to focusing dam breaks in water waves"

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The purpose of this Supplemental Material is to provide some experimental details about Fourier analysis of our experimental data. All figures and references within this document are prepended with "S" to distinguish them from corresponding numbers in the paper.

The Fourier analysis of experimental results presented in Fig. 1(a) of our paper (wavepacket width : 160 s and wave steepness ~ 0.19) is reported in Fig. S1. Fig. S1 shows the Fourier power spectra of the surface elevation measured at $z = 6$ m and at $z = 120$ m. It shows clear evidence of a significant frequency down-shifting already reported e.g. in ref. [S 1]. This frequency down-shifting correlated with a significant spectral broadening represents the clear signature that the dynamics observed at long evolution distance ($z > 50$ m) in Fig. 1(a) of our paper is influenced by high-order nonlinear effects.

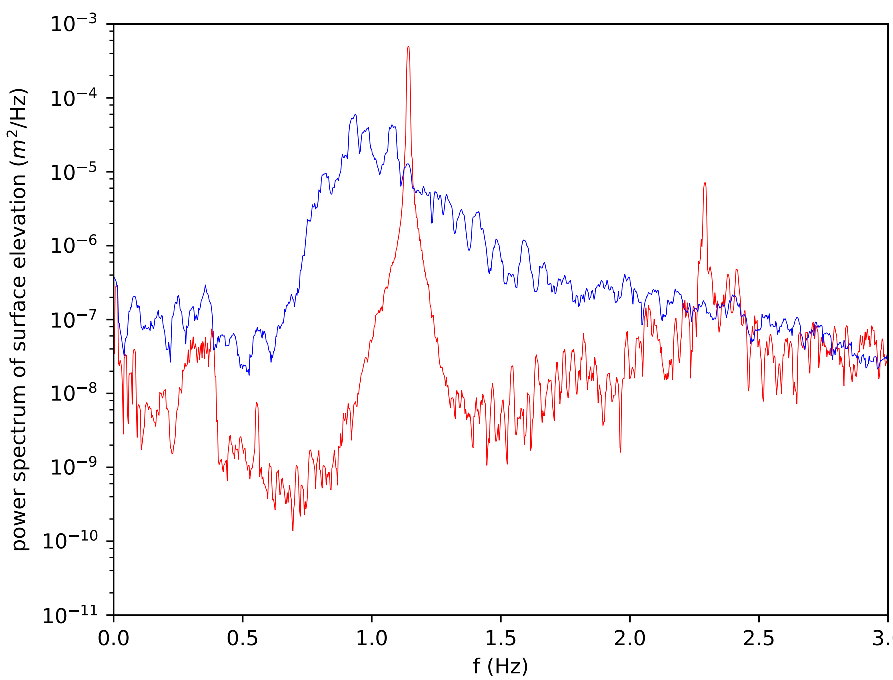

FIG. S1: Power spectrum of surface elevation computed at the 1st gauge (red line) located at a distance $z = 6$ m from the wavemaker. Power spectrum of surface elevation computed at the 20th gauge (blue line) located at a distance $z = 120$ m from the wavemaker. All data are computed from Fig. 1(a) of our paper.

Fig. S2(a) is identical to Fig. 6 of the paper. It shows the space-time evolutions of the three rectangular envelopes that are observed in the situation where the strength of the nonlinearity is increased as compared to Fig. 1(b). Fig. S2(b) shows the power spectra of the rectangular wave packet having a duration $\Delta T_2 = 45$ s (central box in Fig. S2(a)) at $z = 6$ m (red line) and at $z = 120$ m (blue line). Even though frequency down-shifting is not observed in the studied regime, the asymmetric space-time evolution in Fig. 6(a) is associated with a significant spectral broadening phenomenon. In particular the carrier wave at $f_0 = 1.28$ Hz is surrounded by two side bands which represent the spectral signature of the strong modulation due to the DSWs, see Fig. 7(c) of the paper. The fact that the assymetric space time evolution is not correlated with a measurable frequency down-shift might be due to some local change of the instantaneous carrier frequency at points where the wave envelope reaches high amplitude. The exact understanding of these effects requires a careful analysis which is out of the scope of the present work.

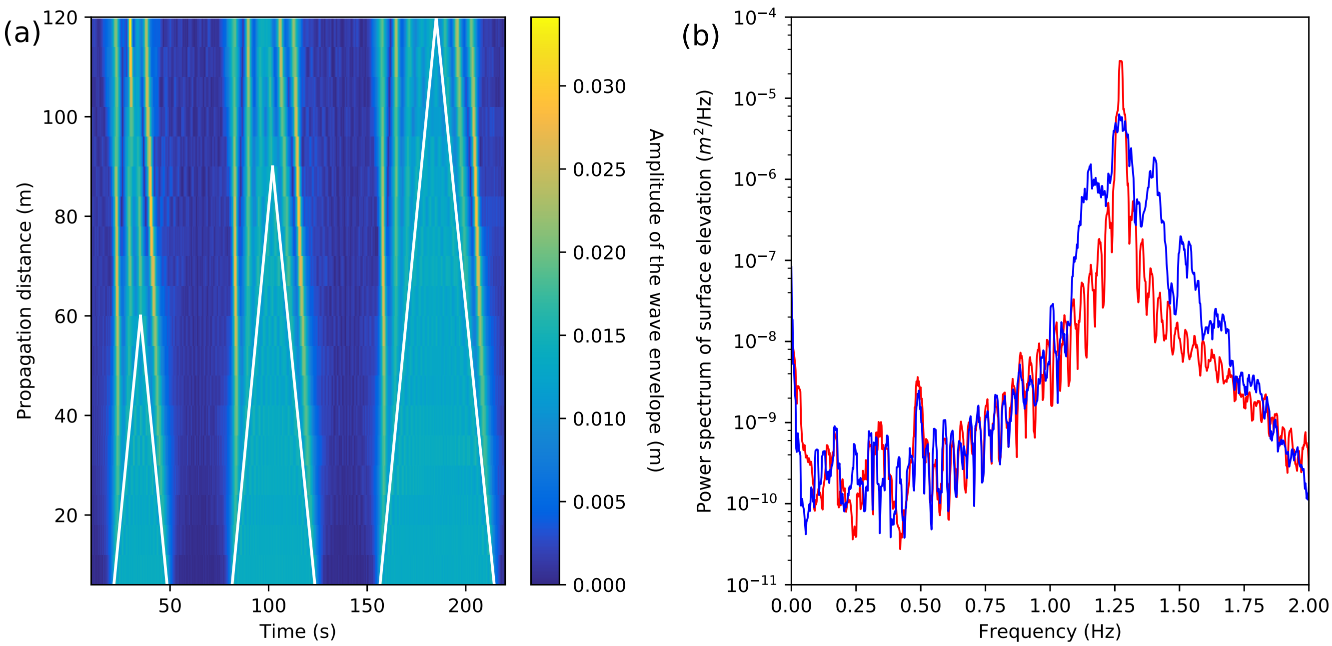

FIG. S2: (a) Space-time evolution of the envelopes of three rectangular wave packets ($\Delta T_1 = 30$ s, $\Delta T_2 = 45$ s, $\Delta T_3 = 60$ s) in the regime where higher-order nonlinear effects have a perturbative influence. The carrier frequency $f_0 = 1/T_0$ is 1.28 Hz. Other experimental parameters are $k_0 = 6.58$ m^{-1}, $a = 0.014$ m (the wave steepness is $k_0 a = 0.09$). (b) Power spectra of the rectangular wave packet having a duration $\Delta T_2 = 45$ s (central box in (a)). The red (resp. blue) line represents the spectrum measured at $z = 6$ m (resp $z = 120$ m). All data are computed from Fig. 6 of our paper.

[S 1] M.-Y. Su, M. Bergin, P. Marler, and R. Myrick, Journal of Fluid Mechanics **124**, 45 (1982).