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Shape of convective cell in Faraday experiment with fine granular materials

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Abstract

The shape of convective cell at the onset of heap formation in the Faraday experiment with fine granular materials is studied in a simple experiment. The motion of fine grains are found to be confined in a single thin conical cell instead of a pair of convective cells as considered previously. The subsequent pattern-selection is strongly affected by the shape and size of the grains as well as the presence of ambient air.

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Large conglomerations of simple grains of macroscopic size, also known as granular materials, exhibit both fluid- and solid-like properties under shear. When subjected to vertical vibration, they show many interesting behaviours including heap formation [1–5,19], surface-instability [6–9,20,21], pattern-formation [10,22,23], segregation of grains of different sizes [11]. For recent and lucid details on the behaviour of granular materials, we refer to Refs. [12,24–26]. Recent experiments [13,27] with thick layers (typically 50–200 layers) of fine grains (particle size ≈ 40–120 μm) vibrated sinusoidally in the vertical plane showed some new patterns – corrugation of heap at low driving frequency f (< 100 Hz) and synchronous stripes at the flattened free surface at high frequency (> 120 Hz). At low vibration frequency and relatively high acceleration amplitude a of driving, a single heap was found to break into many small ones and then self-organize themselves on a hexagonal lattice. The dynamic heap as well as all these patterns disappeared in absence of ambient air surrounding the granular

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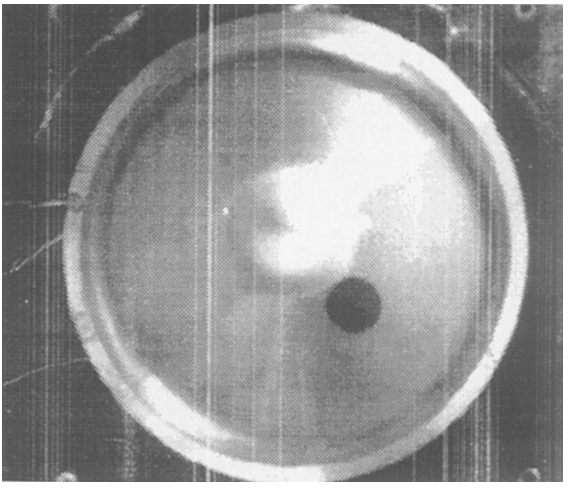

Fig. 1. Top view of a small heap of black test particles near a single white heap of pure alumina in steady state.

material. The role of ambient air in formation of a single heap was first proposed by Faraday [1], and has been clearly demonstrated in experiments by Laroche et al. [3]. This view has been further supported by the experiments of Pak et al. [4]. Faraday suggested that the air current pulls the particles falling at the bottom edge of the heap towards its centre and then fluidized particles move upward to the top of the heap. If it is so, there must be at least a pair of convective cells in the help due to the angular symmetry of the isolated single heap. However, there is no conclusive study on the shape of the convective cell, which is fundamental to the understanding of heap formation, the primary instability of the flat free-surface of a granular material under vertical shaking.

We present here results of our simple experimental study to trace the qualitative structure of the convective cell at the onset of heap formation in fine granular materials of irregular shape. In addition, we study the self-organization of smaller heaps into the hexagonal lattice in more detail. We observe new phenomena in fine granular materials of irregular shape (see Fig. 1 of Refs. [13] for a characterization of grain shape). The wavelength of the hexagonal lattice decreases with increasing frequency, and finally saturates at a finite value as the driving frequency f is raised above a critical value. The saturated value of the wavelength depends on the amount (i.e., the layer thickness) of the material used. The saturation of wavelength has remarkable similarity with the wavelength saturation in other dissipative systems, e.g., vertically excited fluids [14] as well as $2D$ granular layers [15].

The experimental set up is explained in detail in Refs. [13,27]. Small amount of dry and non-cohesive granular materials in a rigid container with flat bottom is vibrated sinusoidally in the vertical plane using an electromagnetic shaker. Various powders (e.g., pure alumina, silica gel and crushed sand) of particle size between 40 and 120 μm are used for this purpose. Results are independent of lateral walls as long as the

amount of the material is small and external parameters, the amplitude and frequency of drive, are chosen such that the material is not completely fluidized. The results reported here are mostly for pure alumina.

To trace the path of moving grains inside the heap, we do three simple experiments. First, we colour a small amount of the same material with black fountain-pen ink and dry it at approximately 200°C for one hour. We begin with a situation when a dynamic heap of pure alumina ($\approx$ 40 g) has reached its steady state, close to the onset of heap-formation. We now place a very small amount ($\approx$ 2–4 g) of the coloured alumina very close to the white heap (see Fig. 1). The coloured particles are sucked into the white heap at the base. We see a black stripe appearing and moving quickly upward along the free-surface and ending near the top of the heap. Some white particles are still falling over this black stripe. We keep vibrating the container for more than 100-times the period of vibration. The stripe becomes slowly grey without having much changes in its thickness. Then we "quench" the heap by making the amplitude of vibration zero very quickly. We then carefully remove particles from the upper part of the heap using a thick paper or a thin aluminium sheet so that we can see the horizontal cross-section of the heap. We see coloured particles only in a thin layer below the grey stripe at the surface. The same result is confirmed by seeing various cross-sections at different heights of the heap. This shows that the coloured particles have remained in a thin stripe parallel to the free surface. The penetration of the coloured particles is only a few particle layers below the free conical surface.

In the second experiment, we spread a thin layer ($\approx$ 1 mm thick) of black particles, in the form of a circular disc, and then place white particles above that to form a conical heap (height $\approx$ 200 mm) such that the black disc is in the central part of the white heap. The heap of white particles is roughly of the same size one would find in the steady state at a frequency (30 Hz in our case) previously decided. This is done when there is no vibration. Now we increase the amplitude of vibration slowly in small steps to $1.2g$ (g is the acceleration due to gravity), which is slightly above the threshold value for heap formation, at the fixed frequency. We leave the container vibrating for very long period so that the heap attains steady state. Now again we "quench" the heap in the similar way as done earlier. Now we cut the cross-sections at various heights and observe the positions of the coloured particles. The cross-sections are found to be white circular layer at all the heights except very close to the base. At the base we see, in stead, black circular sheet surrounded by white circular ring. This suggests that there is almost no motion in the central part of the heap, at least at the onset of heap-formation. There is no noticeable upward motion of grains in the centre of the heap. This is in agreement with recent cinematographic observations of Thomas and Squires [16]. We see horizontal motion of the coloured particles only near the outer edge at the base of the heap. Particles do travel inside the heap when driving amplitude is much above the onset value.

In the last experiment to trace the particle motion, we vibrate white particles to form a heap and reach in steady state. We now set the amplitude of vibration quickly to zero. We make a thin layer of black particles in form of a ring at the base of the heap

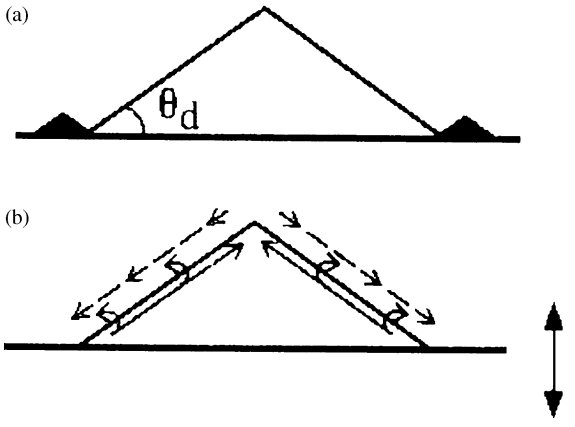

Fig. 2. Schematic diagram of the particle motion in a dynamic heap of granular materials under vertical shaking. The dynamic angle of repose θ_d is uniform. Side views of the position of test particles before the experiment (a) and during the experiment (b).

(see Fig. 2a for a schematic diagram). We raise the amplitude of vibration slowly above the onset value for heap formation. The black particles slowly disappear and the surface of the heap becomes black with some white particles falling over it. The colour of the free-surface of the heap slowly changes and becomes grey. The angle of dynamic repose θ_d remains almost uniform during the experiment. We further "quench" the heap after vibrating it sufficiently long. The cross-sections of the heap everywhere show white circular layers surrounded by grey rim stripe. At the base, we find in stead white circular layer surrounded by grey rim stripe. This along with above two observations suggest that the convective motion of the moving particles in the heap due to the fluidization is confined in a single thin conical cell rather than the presence of a pair of convective cells as previously reported (e.g., see Fig. 5c of Ref. [3]) or many rolls as one finds in the problem of thermal convection in fluids [17]. Fig. 2b shows the flow of coloured particles schematically. The results with crushed sand or silica gel are similar except that the heap for silica gel is almost spherical rather than conical.

We now focus our attention to the vibration-frequency range between 20 and 50 Hz, where hexagons synchronous with the driving frequency are observed. When we increase the driving amplitude, at a fixed frequency, slowly much above the threshold for heap formation, we see the slope of the single heap breaking spontaneously into two, then travelling surface waves appearing from the resulting kink and corrugation of the single heap [13,27]. All these instabilities appear with fine grains of irregular shape and in *presence* of ambient air. The role of shape and size of bigger grains in the spontaneous stratification in granular materials during avalanches is known [18,28]. In the convective layer, which is in the form of a thin conical cell, dissipation may be large due to inelastic collisions and inter-granular friction. Further raising the driving amplitude, the thickness of the convective cell increases and the single heap cannot be

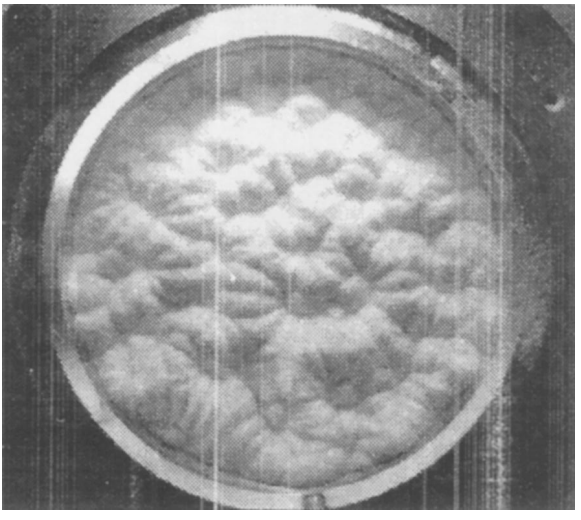

Fig. 3. A typical case of interacting heaps at $a = 5.5g$ (g is the acceleration due to gravity), $f = 42$ Hz for 40 g of alumina showing a spatio-temporal pattern.

supported on the fluidized base. It collapses and give rise to many smaller heaps of similar sizes. These heaps interact with each other at their bases. The particles falling along the free surface of a heap may be sucked into the other. This leads to coalescence of two heaps. Similarly, a bigger heap is unstable due to more fluidization effect and breaks into two smaller heaps. These lead to irregular spatio-temporal patterns (see Fig. 3 for such a situation).

These heaps keep interacting with neighbouring heaps and spontaneously self-organize themselves into a hexagonal lattice with further increase in excitation amplitude. Perhaps, space-filling hexagonal arrangement from heaps of almost circular cross-sections cost the least amount of energy. Fig. 4 shows a typical hexagonal pattern. The hexagons are synchronous with the driving frequency and they disappear in absence of ambient air. We see, instead, subharmonically generated patterns [10,22,23]. We do not see these synchronous hexagons in similar experiments with bigger spherical grains (diameter ≈ 1 mm) and in presence of ambient air.

Fig. 5 shows the variation of the wavelength of hexagons as a function of excitation frequency. The wavelength of hexagons decreases with increasing frequency and ultimately saturates at a finite value. This behaviour is similar to the saturation of pattern-wavelength in Faraday experiment with liquid–vapour system [14] as well as in a thin layer of granular materials [15]. The dissipation in our granular system is enhanced due to inter-particle inelastic collisions and friction. At fixed values of f and a, the wavelength increases with the amount of granular material in the container. The saturated value of the wavelength also shows the similar behaviour (see Fig. 5). The critical value of frequency at which the wavelength of hexagons saturates depends weakly on the amount of the granular material.

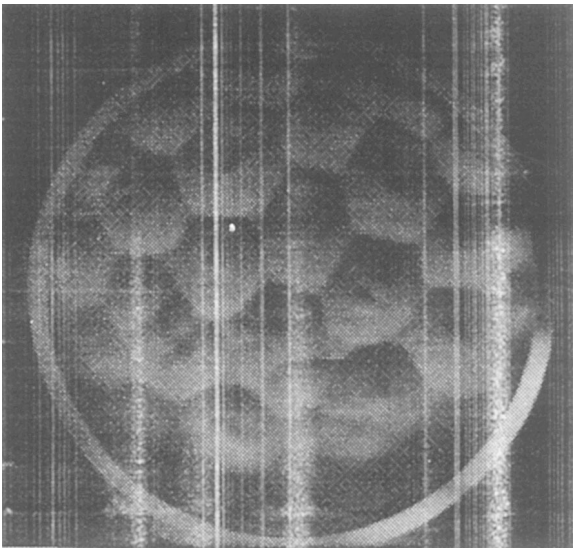

Fig. 4. Heaps self-organize themselves in the form of hexagons at $a=4.26g$, $f=28$ Hz for 40 g of alumina.

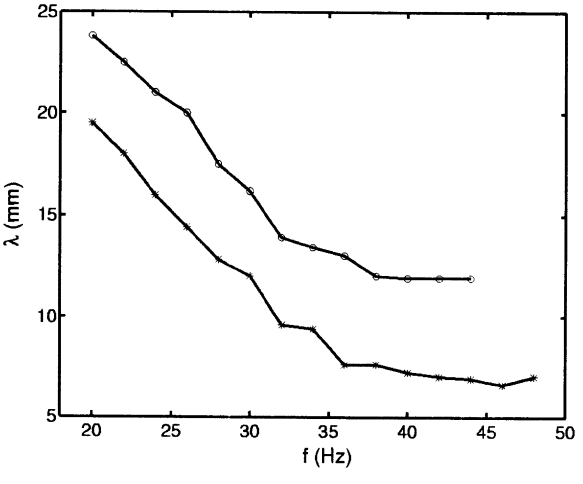

Fig. 5. Dispersion diagram for hexagons for 20 g (symbol ○) and 10 gms (symbol ∗) of pure alumina. The line joins the experimental points.

The threshold for the onset of hexagonal lattice also depends on the driving frequency and amount of grains (see Fig. 6). For higher frequency, the threshold is higher as in any pattern-forming dissipative system. For larger amount of grains, the threshold is higher.

In this article we have reported new insight about the shape of convective cell in Faraday experiment with fine granular materials. The convective cell, at the onset of heap-formation, is a single thin conical layer including the free surface of the heap.

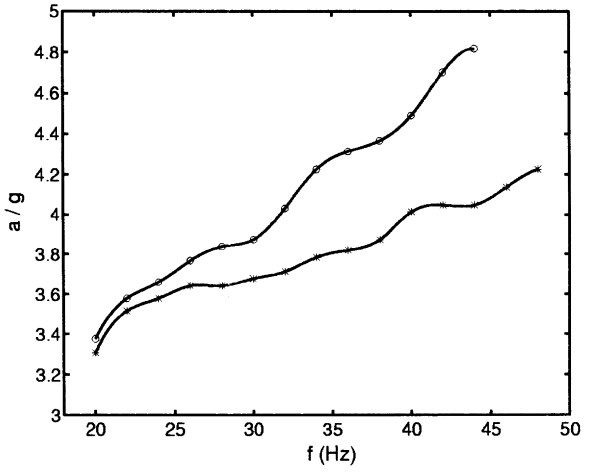

Fig. 6. The threshold (in units of g) for hexagonal lattice as a function of frequency f for 20 g (symbol ○) and 10 gms (symbol ∗) of pure alumina. The line is only a guide to eyes.

Relatively strong fluidization of the granular material leads to breaking of a single heap into many smaller ones, which interact with their neighbours through the contact at their bases. This ultimately leads to hexagons synchronous with the driving frequency. The wavelength of hexagons decrease with increasing frequency and ultimately saturates to a finite value. The presence of ambient air and irregular shapes of fine grains appear to be responsible for the secondary patterns-forming instabilities.

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