MHD wave turbulence
numerical results

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Introduction

Incompressible MHD (divu=0) with strong mean field

•**Alfvén waves** (linearizing about mean field B°) :

$$\partial_t z^+ + (B°.\nabla)z^+ = 0 \quad \partial_t z^- - (B°.\nabla)z^- = 0$$

z+=u-b, z^-=u+b propagate along B° in opposite directions

$\omega = \pm \mathbf{k}.\mathbf{B}° = \pm k_x B°$ (phase velocity vanishes when $k \perp B°$)

•**Nonlinear coupling** of the form z^+z^-

=> only between oppositely propagating wave packets

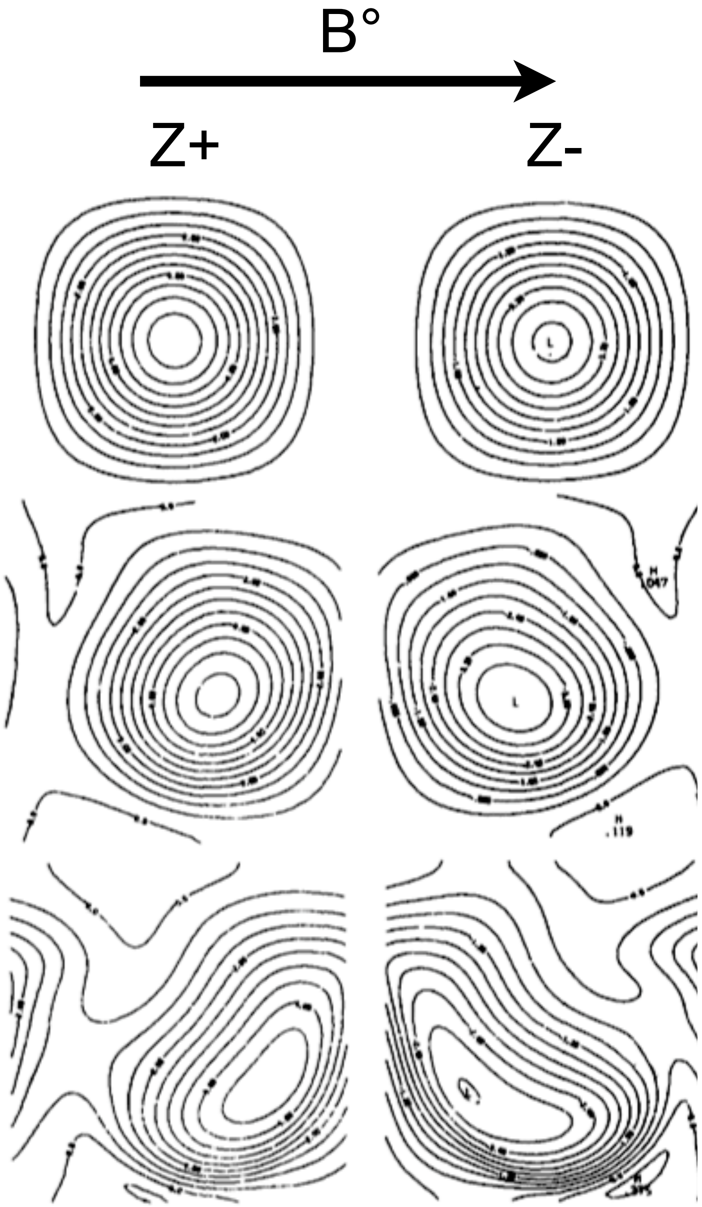
=> *coherent* nonlinear coupling *reduced to transit time*

... *except* when wavevector k in perpendicular direction

- ***idea 1***: perp. direction not important => ***slow*** cascade *Iroshnikov-Kraichnan* 64,65
- ***idea 2***: perp direction dominates=> ***fast*** cascade *Goldreich-Sridhar* 95,98

•Question 1: when mean field B°=0, but still $b \approx u$ (equipartition holds), can we write $B° \approx$ Brms (local mean field)?

•Question 2: is $\delta b/B°$ an important parameter?



Example

Z+ = u-b
Z- = u+b

wave propagation
+ nonlinear coupling

Arguments for a cascade perpendicular to B° (2D case)

$$\mathrm{curl}\,\mathbf{z}^{\pm} = \mathrm{curl}(\mathbf{v} \pm \mathbf{b}) = \omega^{\pm}\hat{\mathbf{e}}_z ,$$

where $\hat{\mathbf{e}}_z$ is a unit vector perpendicular to the $\mathbf{v},\mathbf{b}$ plane. Taking zero viscosity and magnetic diffusivity, we obtain the equations

$$\left(\frac{\partial}{\partial t} \mp i\mathbf{k}\mathbf{B}^0\right)\omega_{\mathbf{k}}^{\pm} = \iint d^2\mathbf{p}\, d^2\mathbf{q}\, M_{\mathbf{kpq}}\,\omega_{\mathbf{p}}^{\pm}\omega_{\mathbf{q}}^{\mp} , \qquad (2a)$$

with

$$M_{\mathbf{kpq}} = \delta(\mathbf{k} - \mathbf{p} - \mathbf{q})(p_x q_y - p_y q_x)(\mathbf{k}\cdot\mathbf{p})/(p^2 q^2) . \qquad (2b)$$

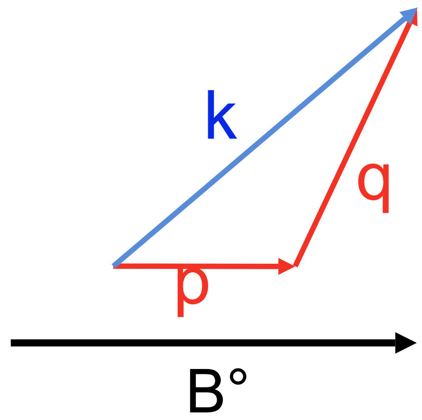
Neglecting the nonlinear terms in the rhs of Eq. (2), we obtain linear Alfvén waves:

$$\omega_{\mathbf{k}}^{\pm}(t) = \psi_{\mathbf{k}}^{\pm} e^{\pm i\mathbf{k}\mathbf{B}^0 t} . \qquad (3)$$

Using the notation of Eq. (3), Eq. (2) gives the following (exact) equation for the waves' amplitudes:

$$\frac{\partial}{\partial t}\psi_{\mathbf{k}}^{\pm} = \iint d^2\mathbf{p}\, d^2\mathbf{q}\, e^{\pm 2i\mathbf{q}\mathbf{B}^0 t} M_{\mathbf{kpq}}\,\psi_{\mathbf{p}}^{\pm}\psi_{\mathbf{q}}^{\mp} , \qquad (4)$$

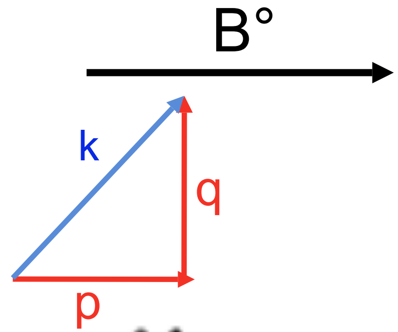
Grappin 1986



kernel averages to zero *except* when q⊥B°

=> Cascade occurs *only* ⊥B°

.. and against a full perpendicular cascade



Born expansion for Alfvén wave collision

when $t \rightarrow \infty$:

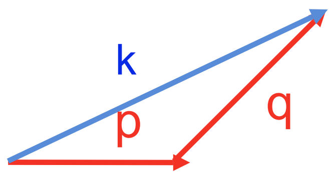
$\omega^{\pm}_{k}(t)$ =

forbids cascade along B°

$$e^{\pm i k B^0 t}\Big(\Omega_{\mathbf{k}}^{\pm} + \frac{\pi}{B^0}\iint d^2\mathbf{p}\, d^2\mathbf{q}\, M_{\mathbf{kpq}}\,\delta(q_\parallel)\,\Omega_{\mathbf{p}}^{\pm}\Omega_{\mathbf{q}}^{\mp} + \frac{\pi}{2(B^0)^2}\iint d^2\mathbf{p}\, d^2\mathbf{q}\iint d^2\mathbf{p}'\, d^2\mathbf{q}'$$

$$\times\big[\pi M_{\mathbf{kpq}}\,\delta(q_\parallel)\,M_{\mathbf{pp'q'}}\,\delta(q'_\parallel)\,\Omega_{\mathbf{p'}}^{+}\Omega_{\mathbf{q'}}^{-}\Omega_{\mathbf{q}}^{-} + iM_{\mathbf{kpq}}M_{\mathbf{qp'q'}}\,\delta(p'_\parallel)\,\Omega_{\mathbf{p}}^{+}\Omega_{\mathbf{p'}}^{-}\Omega_{\mathbf{q'}}^{+}/(q'_\parallel)$$

$$- iM_{\mathbf{kpq}}M_{\mathbf{pp'q'}}\,\delta(q_\parallel + q'_\parallel)\,\Omega_{\mathbf{p'}}^{+}\Omega_{\mathbf{q'}}^{-}\Omega_{\mathbf{q}}^{-}/(q'_\parallel)\big] + O(\Omega^4)\Big).$$



does NOT forbid cascade along B°

Conclusion: in 2D MHD, higher order terms ***do not*** prevent cascade *parallel* to B° (*Grappin, 1986*)

2D MHD: fast or slow cascade?

- 2D MHD with mean field within plane => **no** *true* cascade (no finite time singularity)

- 2D MHD with **no mean field in plane** => *true cascade, power law spectra*

1. **Total energy (E_T=Ev+Em) : -3/2 slope** (*Pouquet Sulem 1988; Biskamp Welter 1989*)
Signature of slow cascade (wave turbulence) of *Iroshnikov&Kraichnan (1965);*
with *energy transfer time:*

$$\tau^* = \tau_{nl} \times (\tau_{nl}/\tau_A) >> \tau_{nl} \qquad (1)$$

and *wave decorrelation time = isotropized Alfvén time* :

$$\tau_A \approx 1/(kB°) \qquad (2)$$

where B° = *local mean field* = *rms field* = rms Alfvén velocity.

2. **Residual energy (E_R=Em-Ev) : -2 spectral slope** (*Biskamp Welter 1989*)
Signature of "Dynamo-Alfvén" balance *Grappin Léorat Pouquet 1983:*

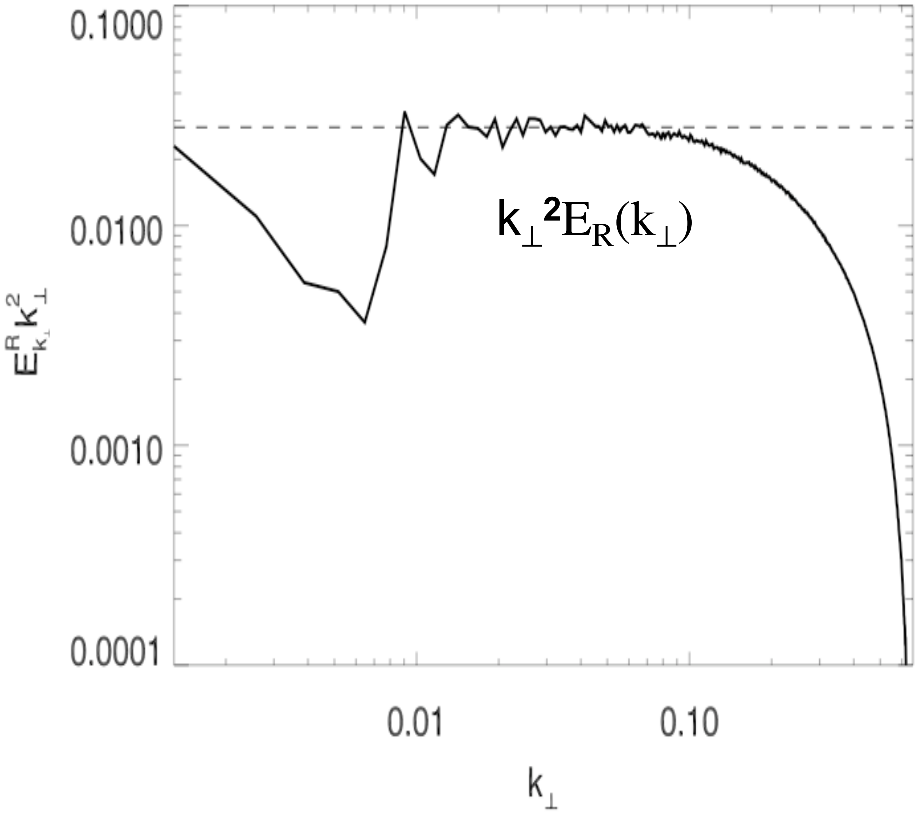
$$E_R(k)/E_T(k) = \tau_A/\tau_* \qquad (3)$$

3D MHD: fast or slow cascade?
a) mean field B°

B°=5b_{rms},10b_{rms} + *forced* turbulence (large scale modes frozen in, E_V=E_M)
=> signature of **Slow cascade**: -3/2, -2 slopes ***in perp plane***
Müller Biskamp Grappin 2004
Müller Grappin 2005

(NB Decaying turbulence
=>different results
Bigot, Politano, Galtier 2008-2009)

Residual spectrum $E_R(k_{perp})$

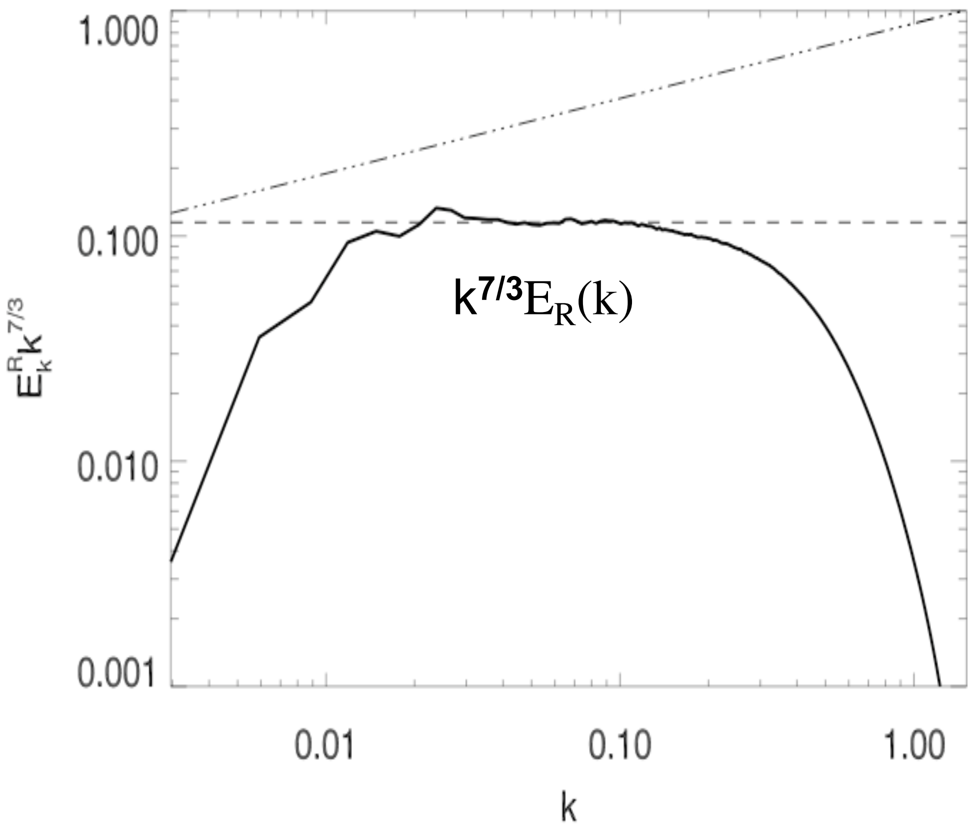


3D MHD: fast or slow cascade?
b) No mean field

Decaying turbulence (starting from $E_V=E_M$ at large scales)
=> signature of **Fast cascade**: -5/3, -7/3 slopes

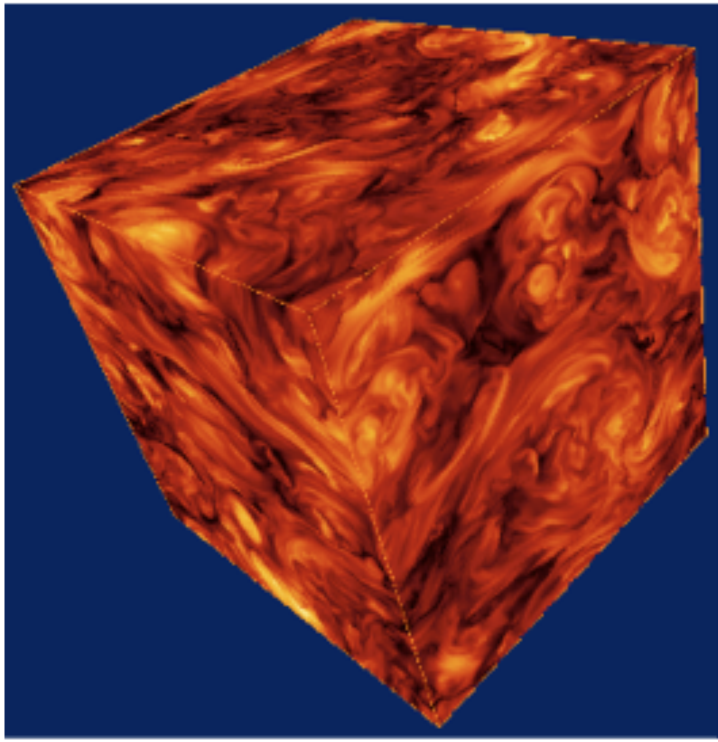
Müller Biskamp Grappin 2004
Müller Grappin 2005

Residual spectrum $E_R(k)$

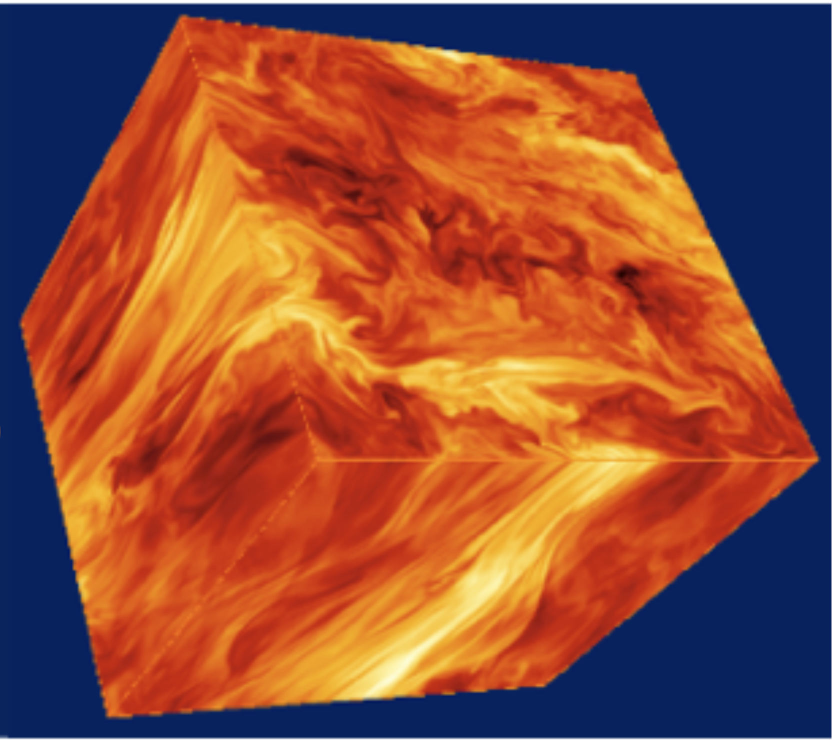


Summary

1024^3, no mean field
decaying



1024^2x512, no mean field
frozen large scales



Isotropic -5/3 turbulence with large magnetic excess

perpendicular (≈2D) IK turbulence, small magnetic excess

3D MHD: comparing the two regimes

(forced) mean field -> (decaying) no mean field

•*big* change in residual energy (-2 -> -7/3): mild -> large *magnetic excess*
•*small* change in total energy (-3/2 -> -5/3): slow -> fast cascade?

Common relation: $E_R(k) = k\,(E_T(k))^2$ (1)

Scenario: Alfvén-Dynamo balance *Grappin Léorat Pouquet 1983 (EDQNM)*:

$dE_R/dt = -E_R/\tau_A + E_T/\tau^* \approx 0$ (2)

with ***slow IK cascade time***: $\tau^* = \tau_{nl} \times (\tau_{nl}/\tau_A)$ (3)

•Alfvén *equipartition time* τ_A defined on mean field B° **or** B_{rms} is B°=0
•Note nonlinear time generalized as $\tau_{nl} = 1/(k(u^2+b^2)^{1/2})$

Case of zero mean field

Can we reconcile:

(1) Alfvén-Dynamo balance scenario

$$dE_R/dt = -E_R/\tau_A + E_T/\tau^* \approx 0$$

with *slow IK cascade time*: $\tau^* = \tau_{nl} \times (\tau_{nl}/\tau_A)$

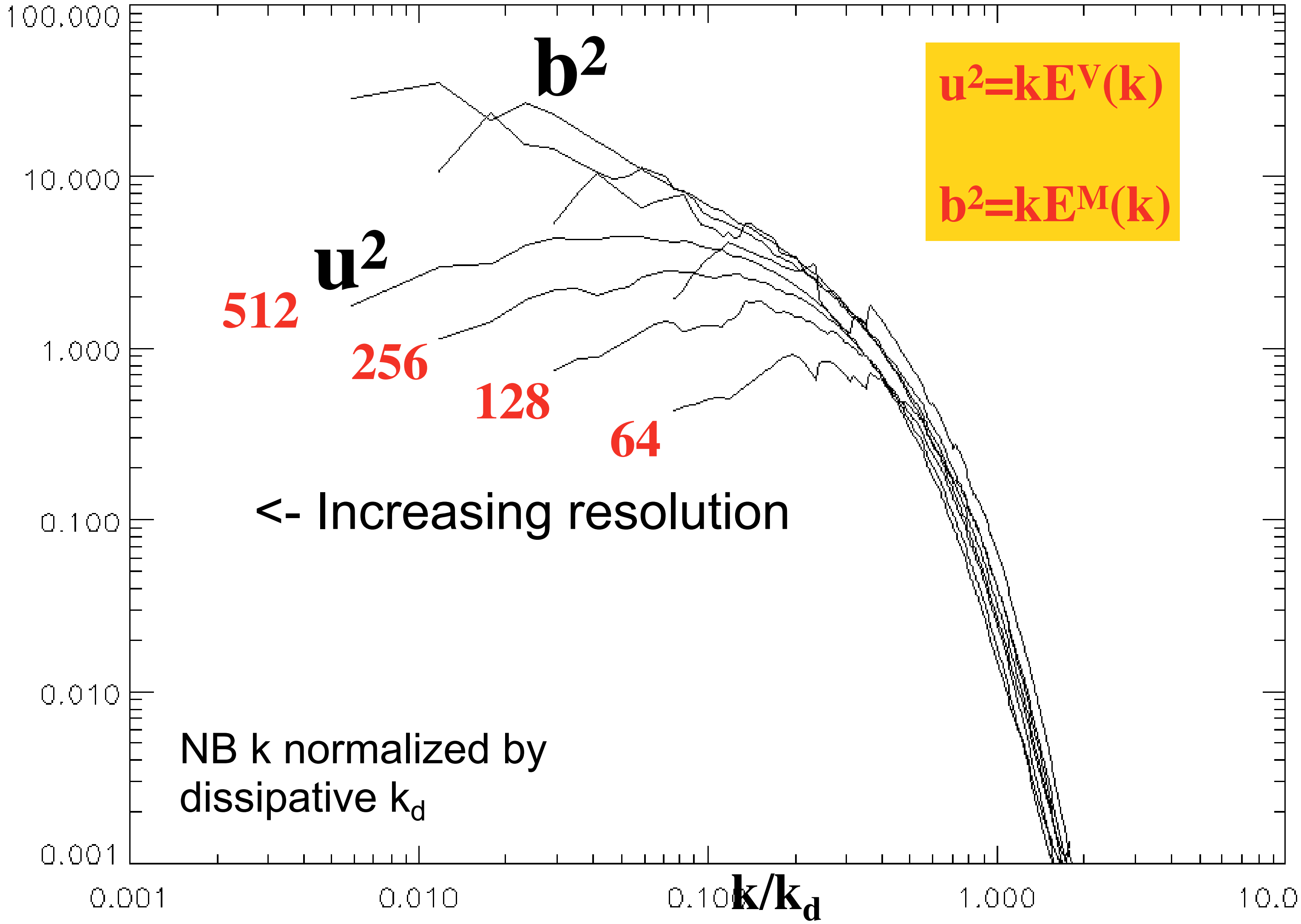
(2) Kolmogorov spectrum for total energy ?

NB Note Kolmogorov spectrum no proof of usual Kolmogorov strong cascade, as:

$$\mathbf{b^2 \gg u^2}$$

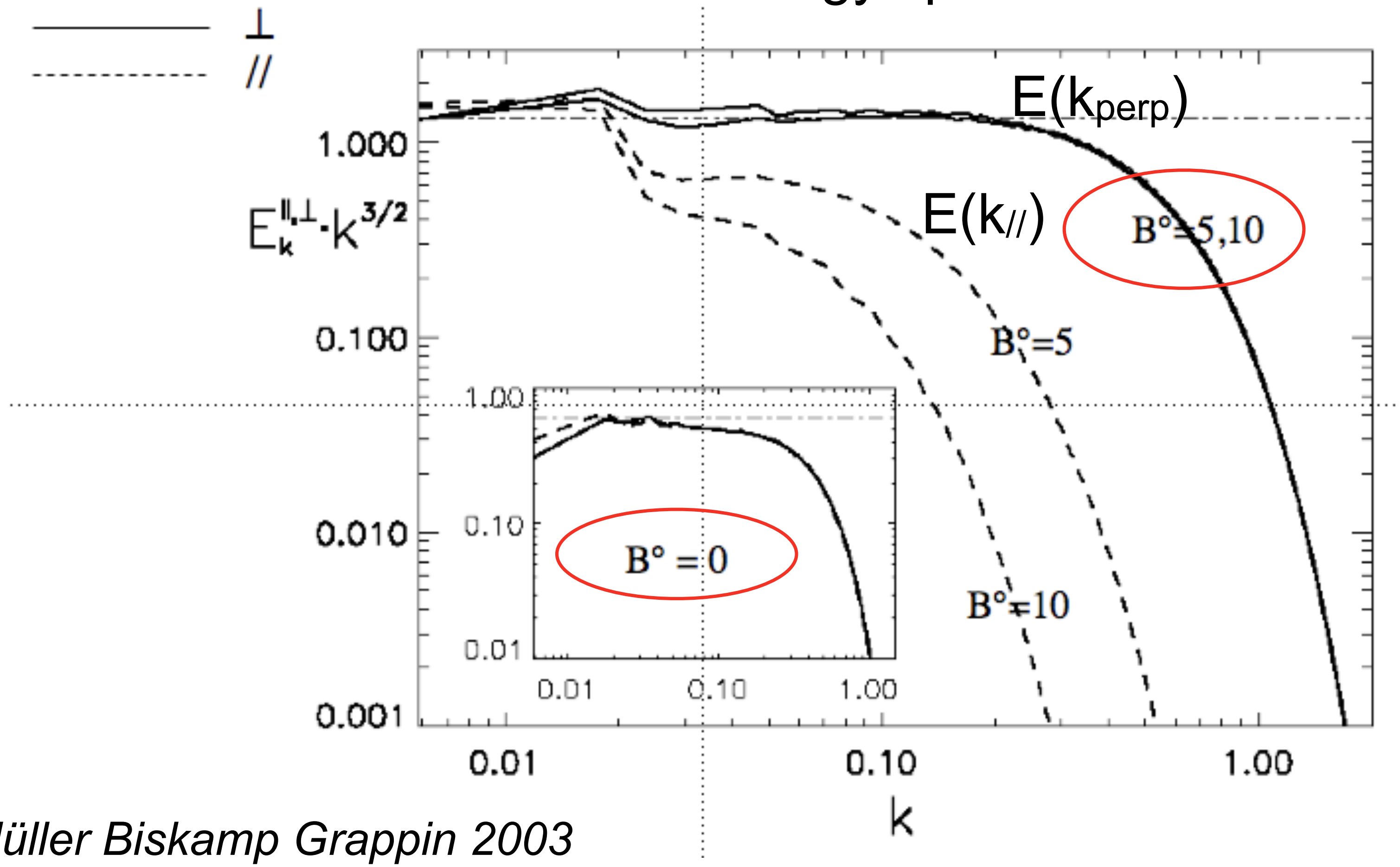
Large magnetic excess

Recall initial conditions: equipartition at large scales



Case of mean field ($B° = 5 b_{rms}$, $10 b_{rms}$)

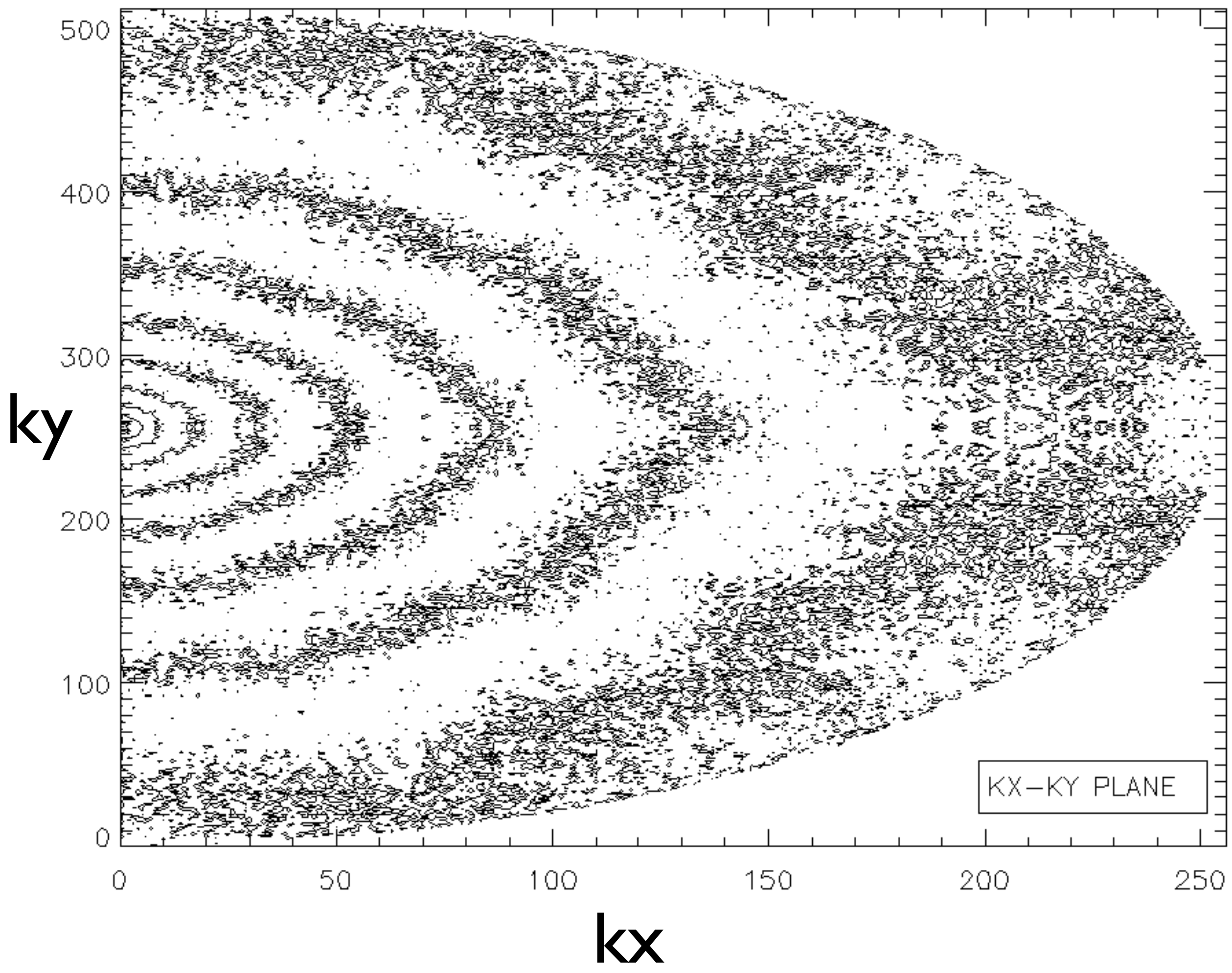
Total energy spectra x $k^{3/2}$



Müller Biskamp Grappin 2003

Mean field (cont.) Isotropy in plane perp to B° (kz=0)

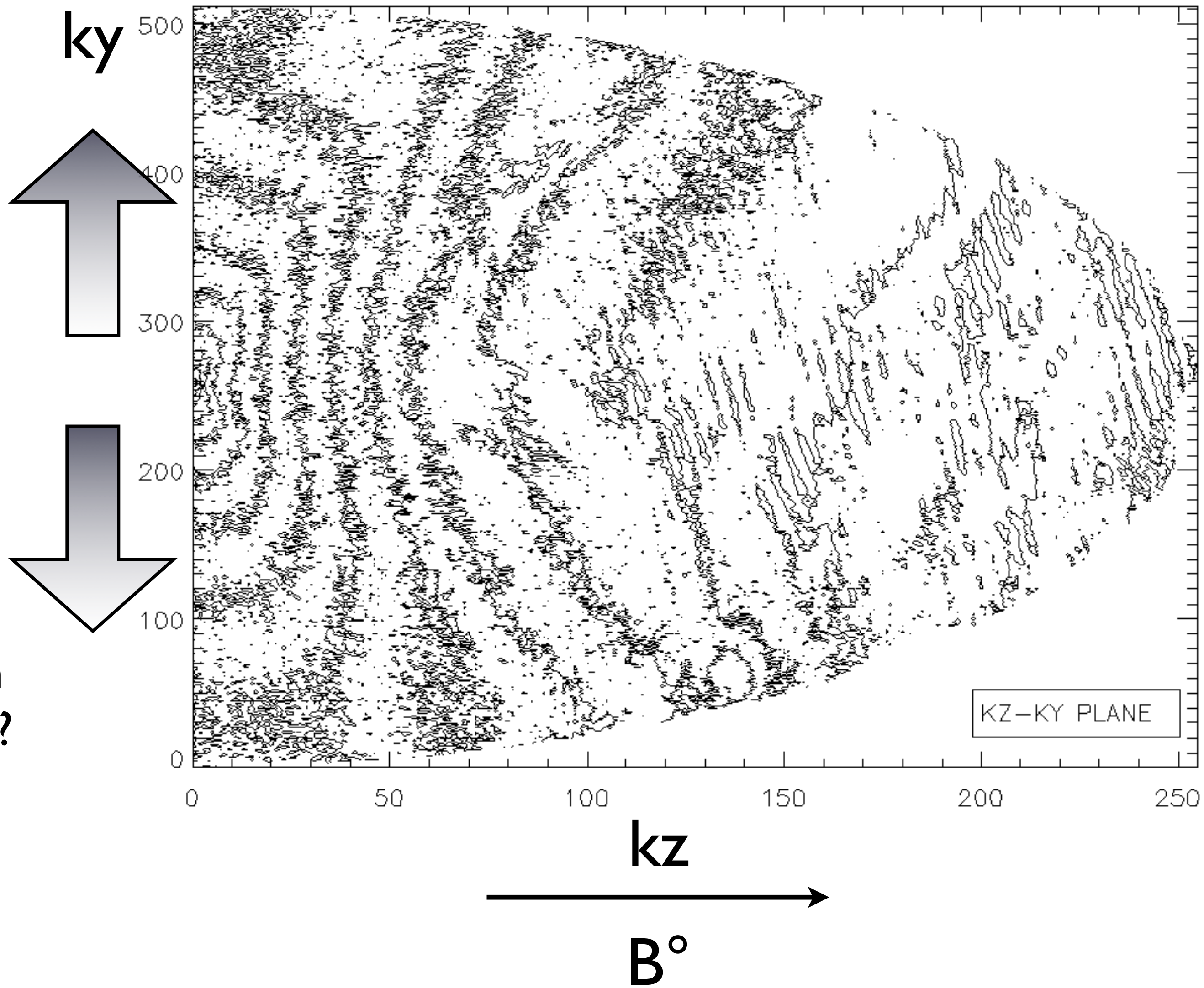
Cutting a (kx,ky) plane at kz=0 through Fourier cube



(Ideal MHD)

Anisotropy in plane Kz, Ky (kx=0)

Cutting a (kz,ky) plane at kx=0 through Fourier cube



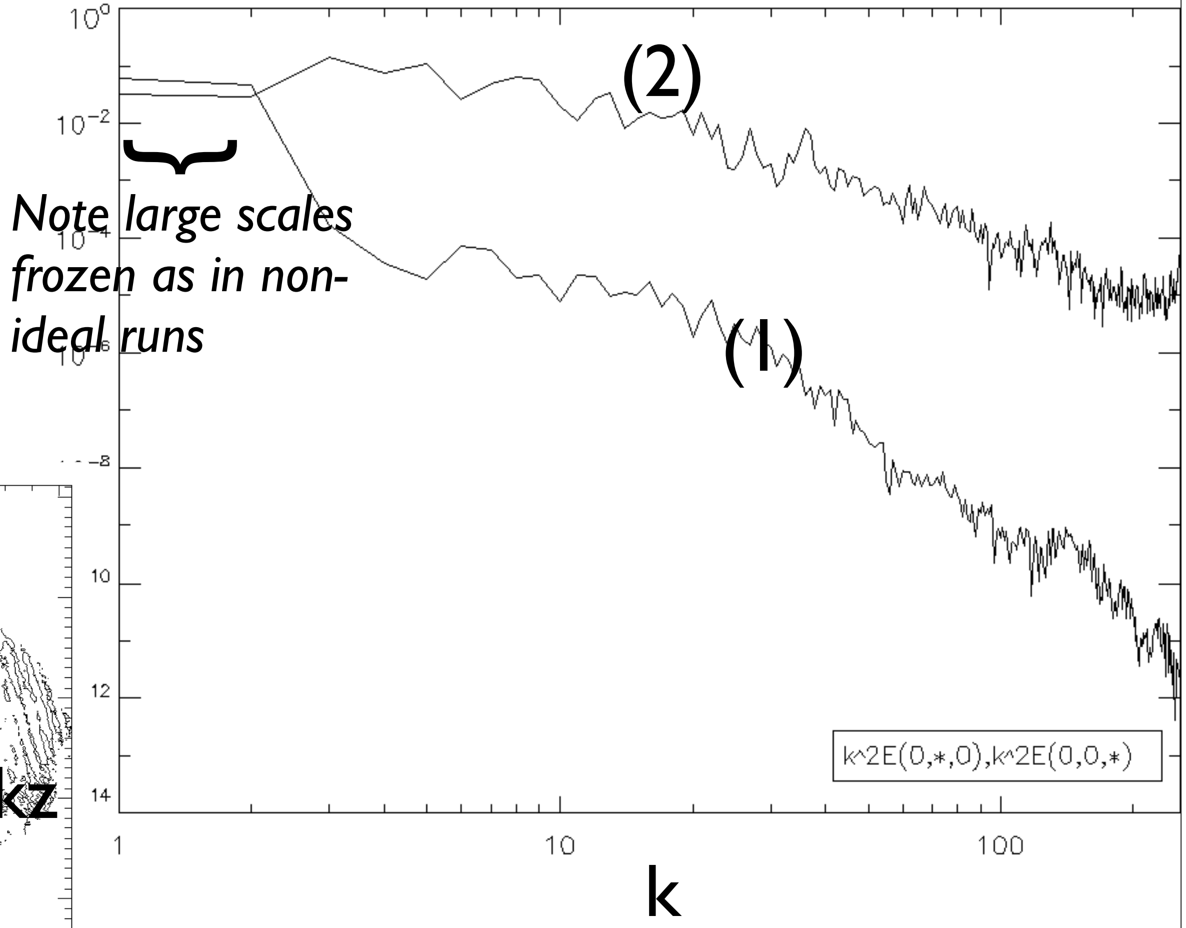
1 Mainly *perpendicular* cascade

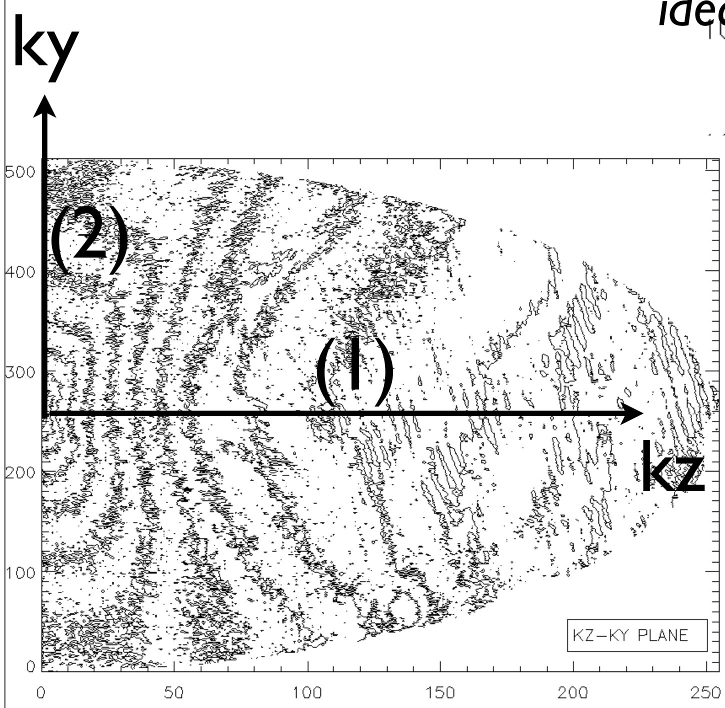
2 is there an inertial range in *parallel direction*?

1D spectra E(ky), E(kz)

Cut the (kz,ky) plane along parallel and perpendicular directions

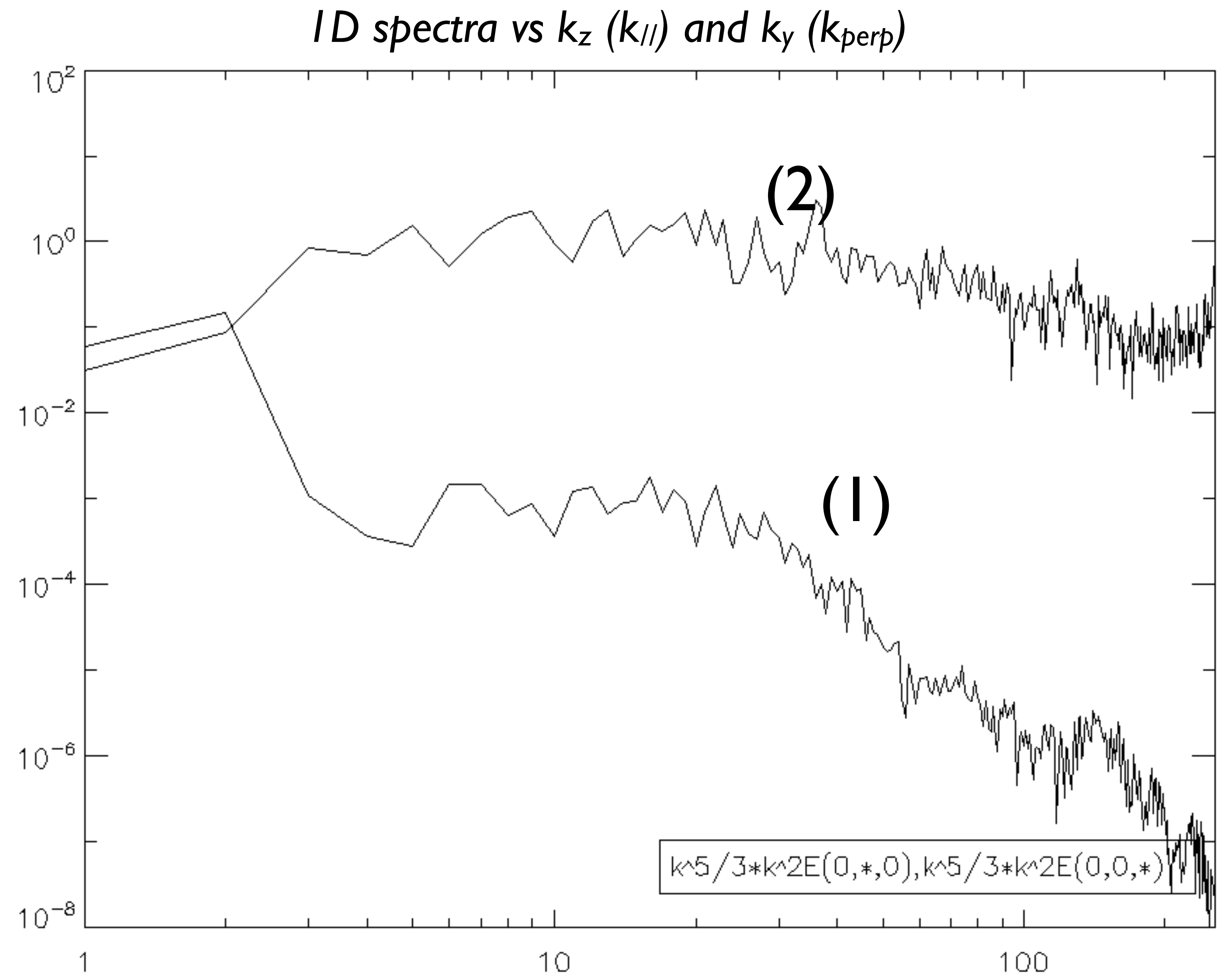
1D spectra vs k_z ($k_{//}$) and k_y (k_{perp})





1D spectra E(ky), E(kz) x $k^{5/3}$

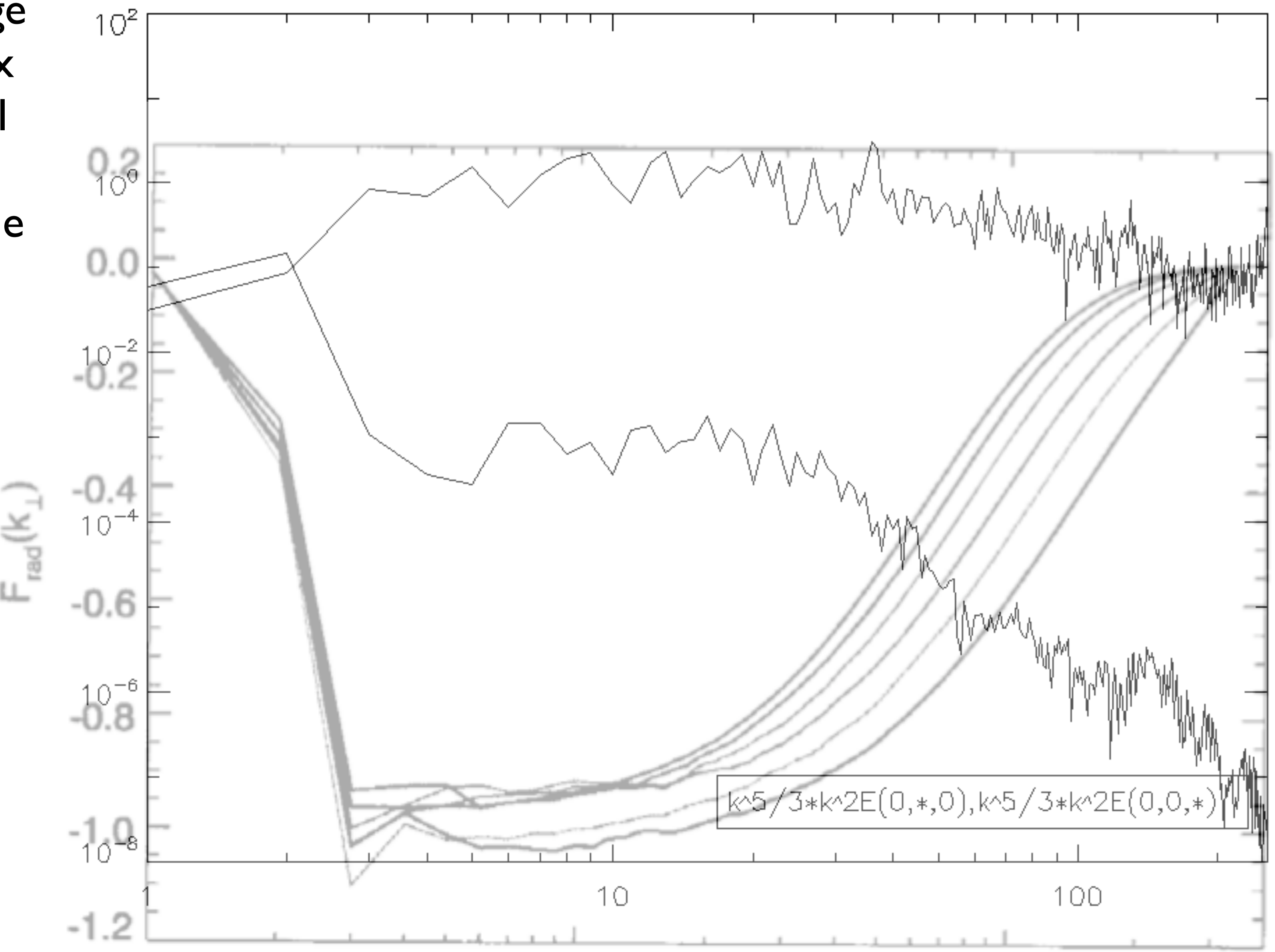
Same but compensated by $k^{5/3}$



Add energy flux to identify inertial range

Quasi-flat range for energy Flux for NON-ideal run shows up in the decade $3 \leq k \leq 10$

Comparison suggests IDEAL inertial range is $3 \leq k \leq 30$



Measuring Anisotropy index q

Method 1: Plot isocontours of $E(k_{//},k_\perp)$

Pick $(k_{//},k_\perp)$ pairs with given energy

=> fit relation $k_{//}(k_\perp)$:

$$k_{//} \approx k_\perp^{\,q}$$

Method 2: use $(k_{//},k_\perp)$ pairs

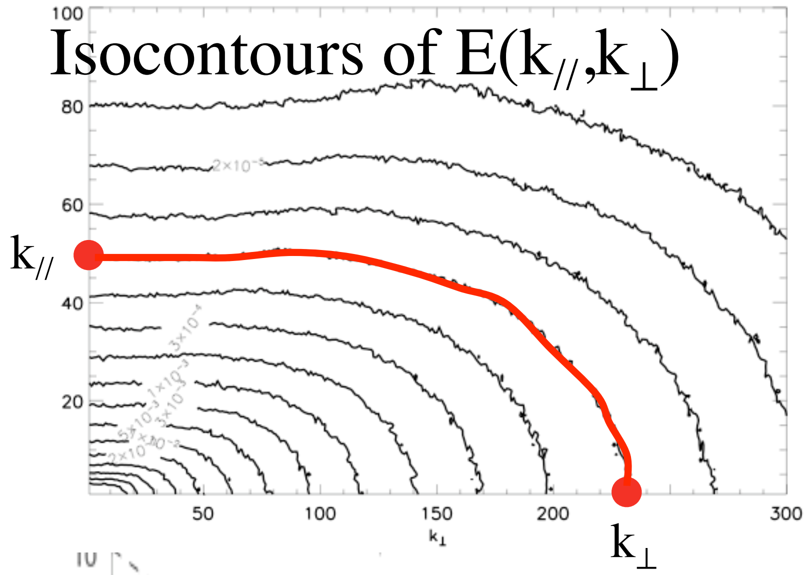
of ***integrated 1D spectra***:

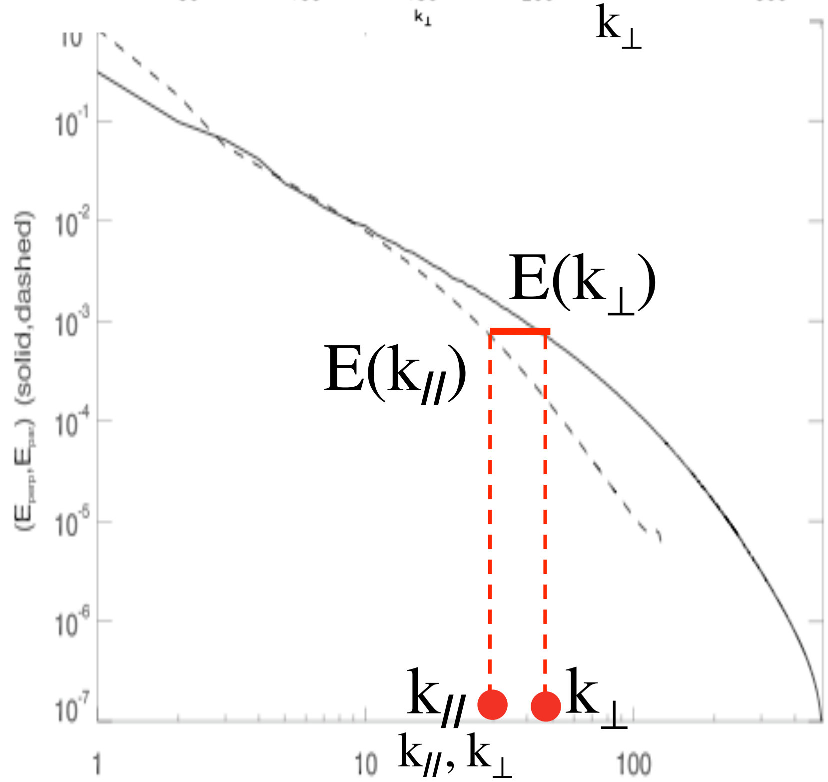
$E_{perp}(k_\perp) = \int dk_{//}\, E(k_{//},k_\perp)$

$E_{par}(k_{//}) = \int dk_{//}\, E(k_{//},k_\perp)$

NB:

$\int dk_{//}\, E_{perp}(k_\perp) = \int dk_{//}\, E_{par}(k_{//})$





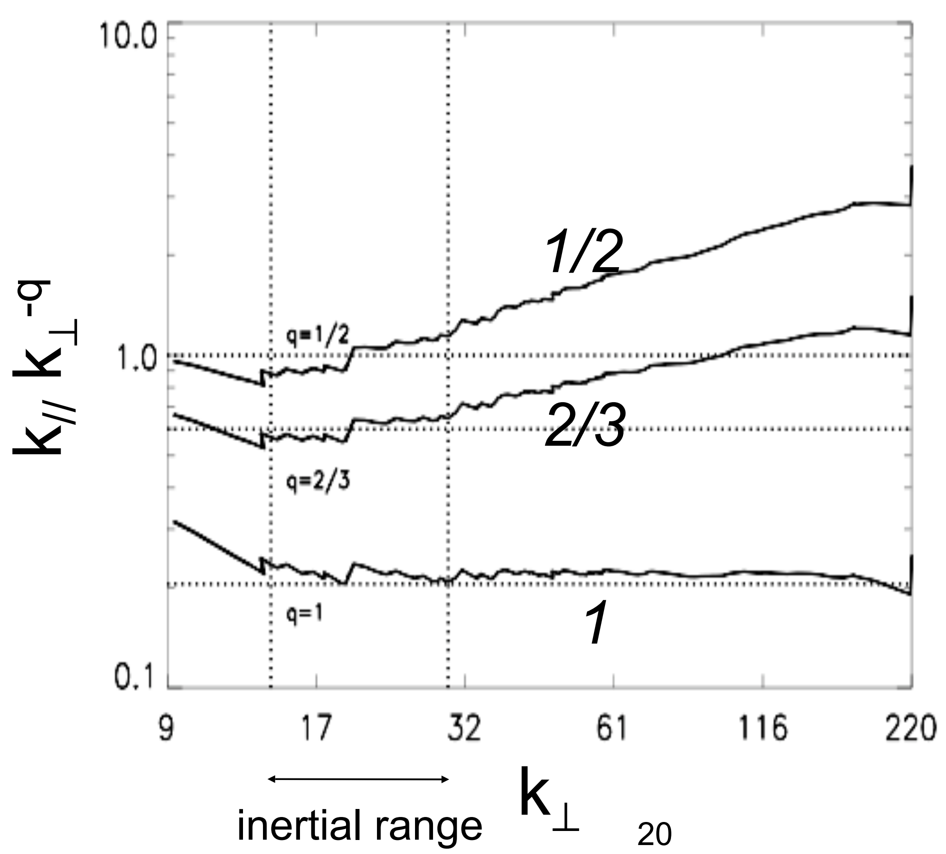
Result (1rst method)

•Global fit for 15<k<200 leads to q=1

•inertial range [15,30] has

1 < q < 2/3

$k_{//} k_{\perp}^{-q}$ vs $k_{\perp}$



Result (2nd method)

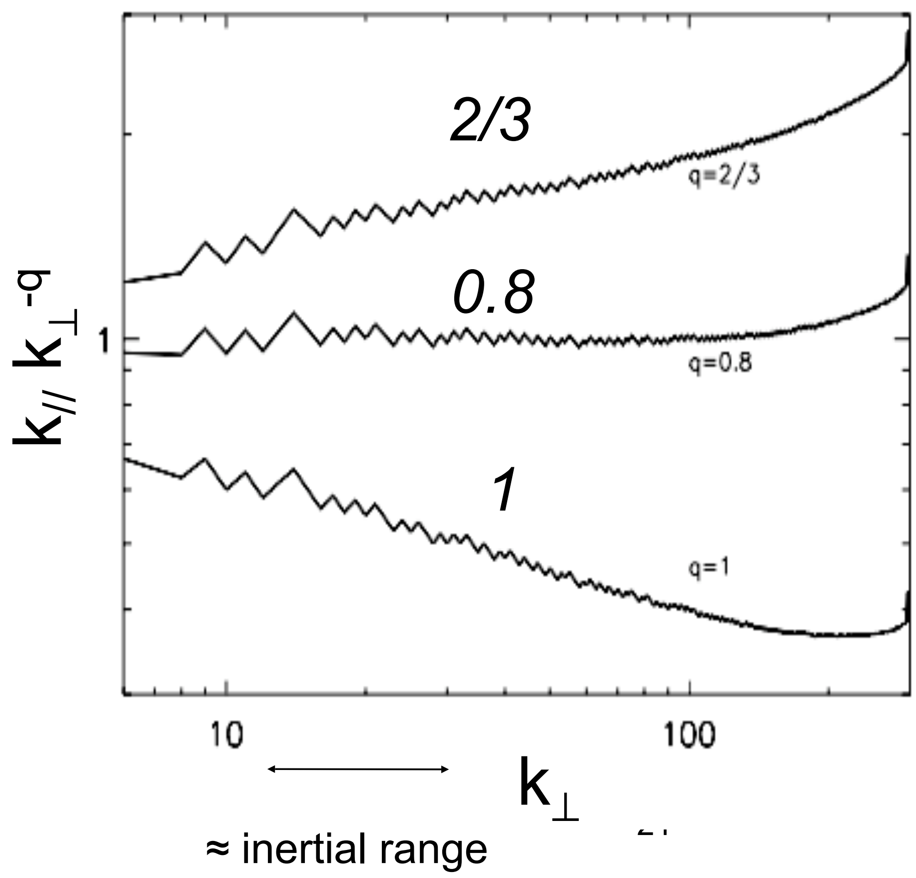
•All scales except very small scales have:

q=0.8

•very small scales are ≈ isotropic :

q≈1

$k_{//}\, k_\perp^{-q}$ vs $k_\perp$



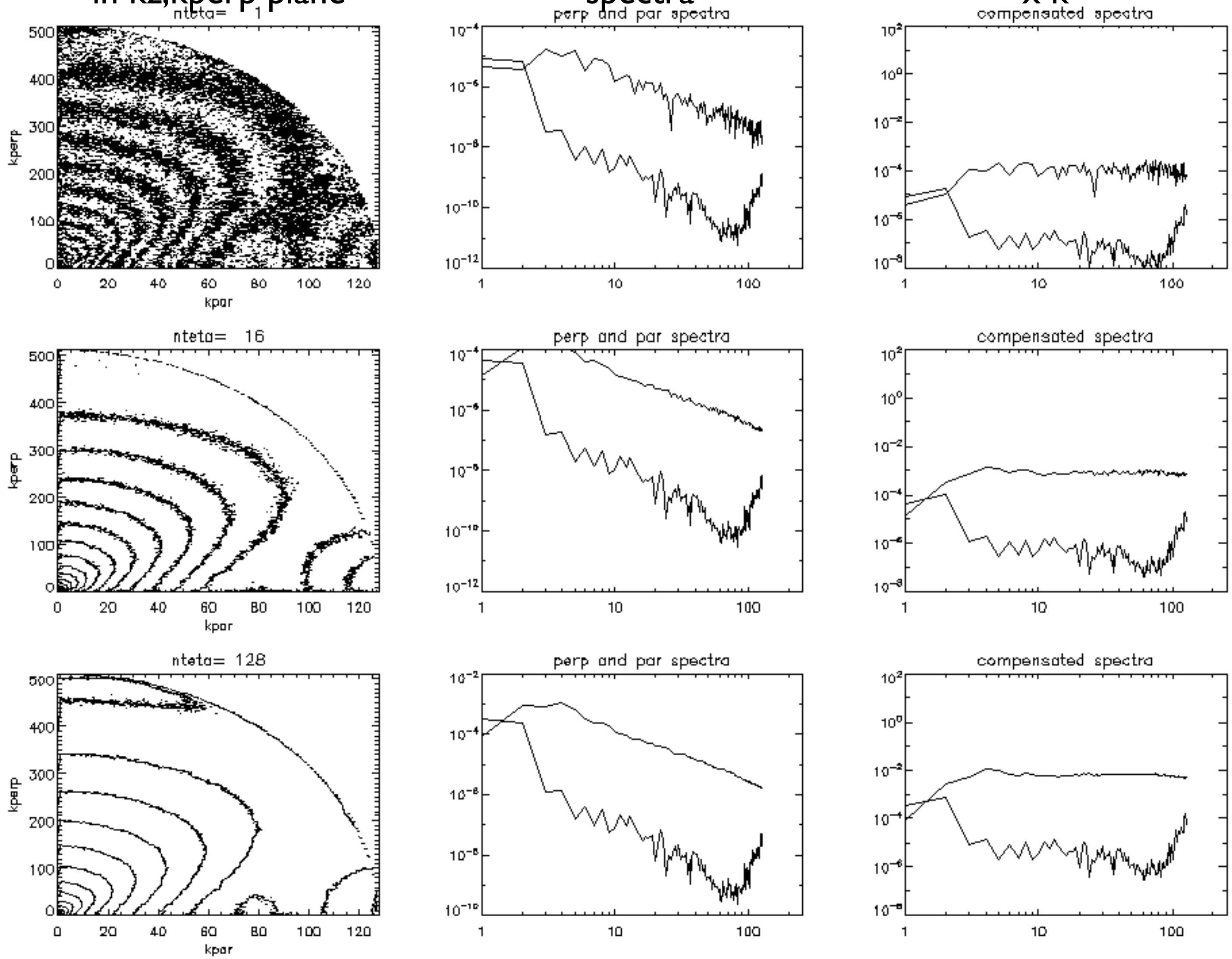
A direct look at anisotropy

Averaging in perp (k_x,k_y) plane

Energy contour in kz,kperp plane

1D perp and par spectra

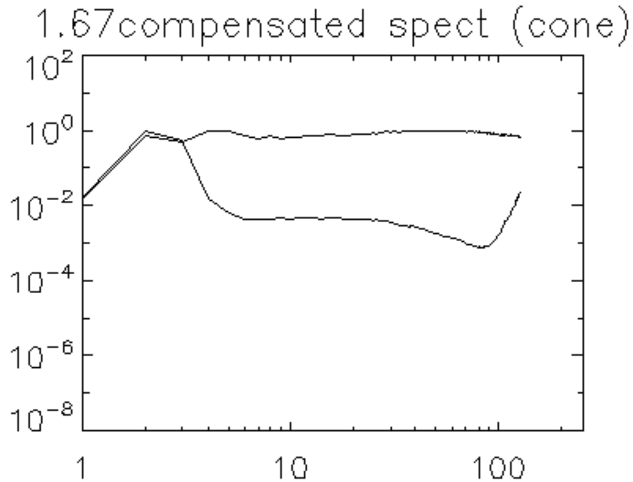
perp& par spectra x $k^{5/3}$

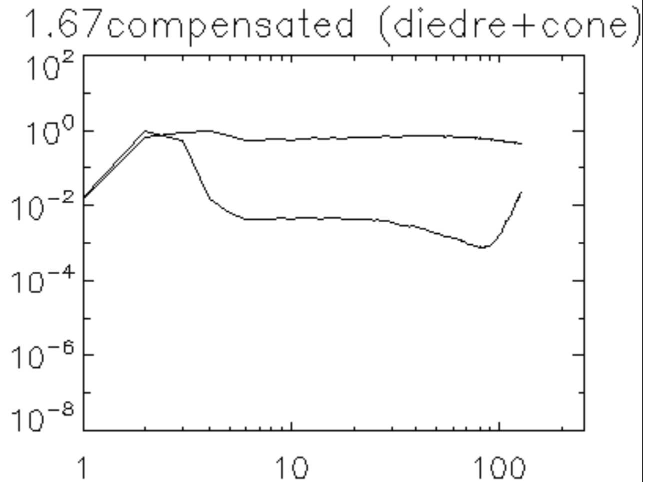


A direct look at anisotropy (cont)

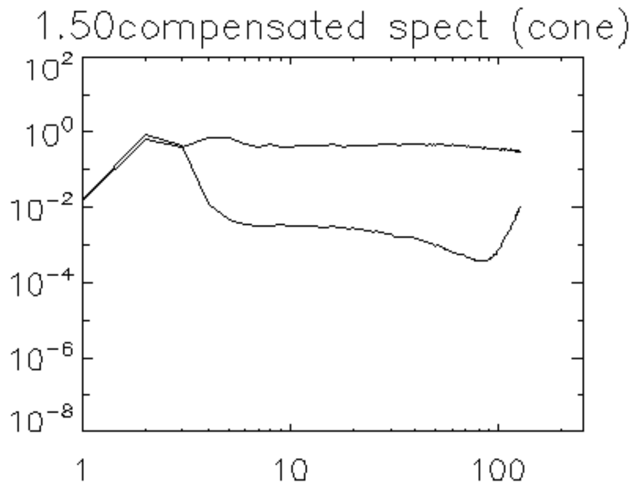
averaging (1) | averaging (2)

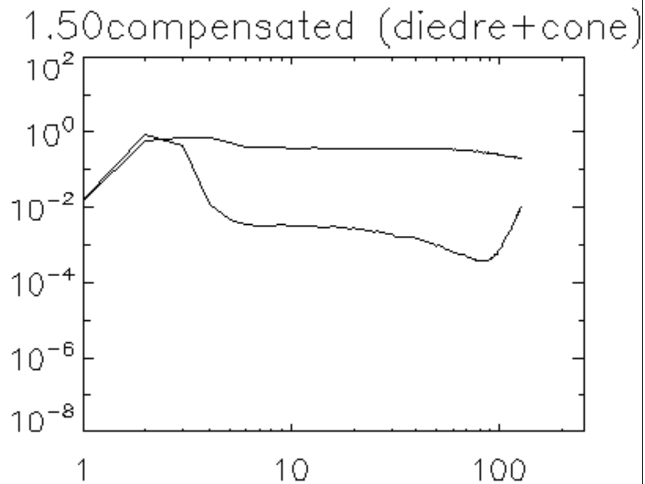
$k^{5/3}$ x spectra





$k^{3/2}$ x spectra





Conclusion

• mean field case, large-scales frozen

Indices for slow cascade in perp plane present in mean field case (see also slow energy decay, in *decaying case Bigot 2008*): dynamo-Alfvén balance holds and analogy with purely 2D MHD.

Note however experimental evidence for IK spectrum only marginal.

Strong anisotropy may be compatible with *GS 1995* (purely passive parallel Alfvén waves). However, direct evidence is also compatible with much weaker anisotropy.

New evidence (work in progress) brought by study of Lagrangian spectra (*Gogoberidze*; *A. Busse, thesis*)

• no mean field, decaying

Fast cascade seems to hold because of -5/3 spectrum, to be reconciled with slow cascade build in (again) nicely verified dynamo-Alfvén balance.